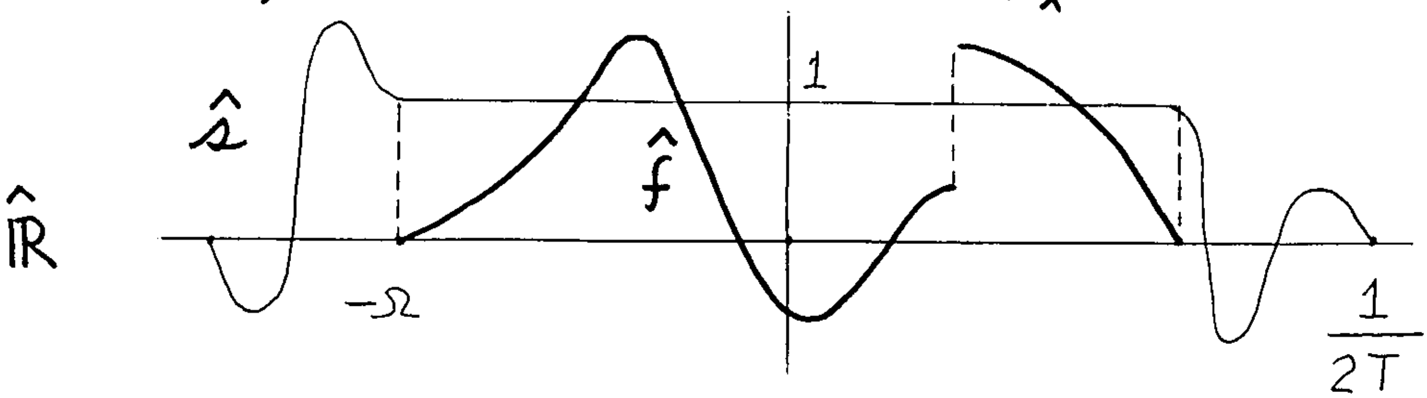
Classical Sampling Theorem

$$PW_{\Omega} = \{ f : \mathbb{R} \to \mathbb{C} \ : \ \|f\|_{L^2(\mathbb{R})} < \infty \ \text{ and } \ \operatorname{supp} \hat{f} \subseteq [-\Omega, \Omega], \text{ i.e., } \hat{f} = 0 \text{ off } [-\Omega, \Omega] \}.$$

__Theorem__ $0 < 2T\Omega \leq 1$, $\quad s \in PW_{1/(2T)}$ such that $\hat{s} = 1$ on $[-\Omega, \Omega]$ and $\hat{s}$ bounded.

$$\forall f \in PW_{\Omega}, \quad f = T \sum f(nT)\, \tau_{nT}\, s$$

uniformly on $\mathbb{R}$ and in L^2-norm. $(\tau_x s(t) = s(t-x))$



__Example__ a. $s_{\Omega}(t) = \dfrac{\sin 2\pi\Omega t}{\pi t}$, $\quad 2T\Omega = 1$.

b. Define $\varphi(t) = (1/\sqrt{2\Omega})\, s_{\Omega}(t)$ and $V_0 = \overline{\operatorname{span}} \{ \tau_{nT}\, \varphi \}$. This gives MRA.

$$\psi(t) = \frac{1}{\sqrt{2\Omega}} \left(s_{2\Omega}(t) - s_{\Omega}(t) \right)$$

is the __Shannon__ __wavelet__.