MATH 241H Dr. Wolfe SAMPLE FOURTH HOUR EXAM FALL 2011

The Use of Calculators Is Not Permitted On This Exam

1. Let $\mathbf{F} = (2xz + y^2)\mathbf{i} + (z^2 + 2xy)\mathbf{j} + (2yz + x^2 + 1)\mathbf{k}$.
(a) Show that F is conservative and find a function f such that $\mathbf{F} = \nabla f$.
(b) Compute $\int_C \mathbf{F} \cdot \mathbf{dr}$ where C is the curve

$$x = t^5, \qquad y = te^{(1-t)} \qquad z = 1 + \cos^3 \pi t/2, \qquad 0 \le t \le 1.$$

2. Compute $\int_C xy\, dx + x^2\, dy$ where C is the boundary of the part of the disk $x^2 + y^2 \le 1$ which lies in the first quadrant, oriented counterclockwise.

3. Use Stokes's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k};$$

C is the triangle with vertices (0, 0, 0), (1, 0, 0) and (0, 1, 1) oriented counterclockwise as viewed from above. (Hint: The triangle is contained in the plane $z = y$.)

4. Compute $\int\int_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS$ where Σ is the boundary of the solid region

$$D = \{(x, y, z) : x^2 + y^2 \le 4,\ -1 \le z \le 3\},$$

$\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^2\mathbf{k}$, and $\mathbf{n}$ points outward.