MATH 241H Dr. Wolfe SAMPLE SECOND HOUR EXAM Fall 2011

The Use of Calculators Is Not Permitted On This Exam

1. Let $w = f(x, y, z)$ be a differentiable function. Suppose that $f(8, -4, 1) = 5$ and $\nabla f(8, -4, 1) = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

(a) Find the directional derivative of f at $(8, -4, 1)$ in the direction toward the origin.

(b) In what direction is the directional derivative of f at $(8, -4, 1)$ a maximum and what is the maximum value of the directional derivative ?

(c) Find an equation of the tangent plane to the level surface $f(x, y, z) = 5$ at $(8, -4, 1)$.

(d) If $x(t) = 8 + 2t, y(t) = 3t - 4, z(t) = e^t$, what is $\frac{dw}{dt}$ at $t = 0$?

2. If $z = e^{-at} \cos ax$, show that

$$\frac{\partial^2 z}{\partial x^2} = a \frac{\partial z}{\partial t}$$

3. Use differentials to obtain an approximate value of $(\sqrt{15} + \sqrt{99})^2$. The exact value is 191.07139547.

4. Let

$$f(x, y) = 3x^2 - 6xy + y^3 - 24y$$

Find <u>all</u> critical points of f. Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

5. The Ace Widget Company has determined that x units of labor and y units of capital can produce $f(x, y) = 60x^{3/4}y^{1/4}$ widgets. Also, suppose that each unit of labor costs \$100 while each unit of capital costs \$200. Assume that \$40,000 is available to spend on production. How many units of labor and how many units of capital should be utilized in order to maximize production ?