AMSC/CMSC 466 FALL 2011 SAMPLE HOUR EXAM I

1. Consider the expression
$$\frac{1}{1-x} - \frac{1}{1+x},$$
assuming $x \neq \pm 1$.
   (a) For what range of values of x is it difficult to compute this expression accurately in floating-point arithmetic ?
   (b) Give a rearrangement of the terms such that, for the range of x in part (a), the computation is more accurate in floating-point arithmetic.

2. Assume a decimal (base 10) floating point system having machine presicion $\epsilon_{mach} = 10^{-5}$ and an exponent range of ± 20. What is the result of each of the following floating-point operations

(a) $1 + 10^{-7}$	(b) $1 + 10^{3}$	(c) $1 + 10^{7}$
(d) $10^{10} + 10^{3}$	(e) $10^{10}/10^{-15}$	(f) $10^{-10} \times 10^{-15}$

3. Let
$$A = \begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix}$$
   (a) Find a lower triangular matrix L such that $A = LL^T$ (Choleski factorization).
   (b) Let $\mathbf{b} = (10, -4)^T$. Use the Choleski factorization to solve $A\mathbf{x} = \mathbf{b}$ by forward elimination and back substitution.

4. In $\mathbf{R}^2$, is it possible to have two vectors x and y such that $\|x\|_1 > \|y\|_1$ but $\|x\|_\infty < \|y\|_\infty$? If so, give an example.

5.
   (a) How is the condition number of a matrix A defined for a given matrix norm ?
   (b) How is the condition number used in estimating the accuracy of a computed solution to a linear system $A\mathbf{x} = \mathbf{b}$?

6. Given the three data points $(-1, 2), (0, 1), (1, 2)$ Find the interpolating quadratic:
   (a) in the form $ax^2 + bx + c$ by solving a system of linear equations.
   (b) in the Lagrange form
   (c) in a Newton form.
   Show that the three representations give the same polynomial.

7. Let
$$s(x) = \begin{cases} x + 1 & -2 \leq x \leq -1, \\ x^3 - 2x - 1 & -1 \leq x \leq 1, \\ x - 3 & 1 \leq x \leq 2. \end{cases}$$
   Is $s(x)$ a natural cubic spline ? Explain.