AMSC/CMSC 466 Dr. Wolfe ASSIGNMENT #3 Due October 25, 2011

1.
   (a) Construct a tridiagonal solver along the lines outlined in class. Your solver should be a function M-file with four vectors as input and the solution as the output. Test your solver on the problem $A\mathbf{x} = \mathbf{b}$ where A is the 9×9 tridiagonal matrix with 2's on the main diagonal and -1's on the super-and subdiagonals and $\mathbf{b}(j) = .01$, $j = 1, \ldots, 9$. As a check, the answer should be $\mathbf{x}(j) = \frac{1}{2}(\frac{j}{10})(1 - \frac{j}{10})$.
   (b) The solution of the boundary value problem

$$-\frac{d^2u}{dx^2} = 4\pi^2 \sin 2\pi x, \qquad u(0) = 0, \quad u(1) = 0$$

   is $u(x) = \sin 2\pi x$. We wish to construct finite difference approximations to $u(x)$ using the tridiagonal solver to find solutions to the system of finite difference equations for $n = 9, 19, 39$. Evaluate the accuracy of your solutions by taking the norm of the difference of the solution vector and the vector of exact solutions values at the nodes.

2. Let $\{x_0, x_1, \ldots, x_n\}$ be $n+1$ distinct points. Let $\{l_j(x), j = 0, 1, \ldots, n\}$ be the corresponding set of Lagrange polynomials. Show that for all x

$$\sum_{j=0}^{n} l_j(x) = 1.$$

3. Ex.1(a), p.143, *Atkinson & Han.*

4.
   (a) Prove that the polynomial of degree $\leq n$ which interpolates $f(x)$ at $n+1$ distinct points is $f(x)$ itself in case $f(x)$ is a polynomial of degree $\leq n$.
   (b) Prove that the k^{th} divided difference $p[x_0, x_1, \ldots, x_k]$ of a polynomial $p(x)$ of degree $\leq k$ is independent of the interpolation points $x_0, x_1, \ldots, x_k$.
   (c) Prove that the k^{th} divided difference of a polynomial of degree $< k$ is zero.

5. For $f(x) = \sinh x$ we are given that

$$f(0) = 0, \;\; f'(0) = 1, \;\; f(1) = 1.1752, \;\; f'(1) = 1.5431.$$

   Find $h_3(x)$, the unique Hermite cubic polynomial interpolating this data. Compare $h_3(0.5)$ with $f(0.5) = .5211$.

6. The Runge function is

$$r(x) = \frac{1}{1 + x^2}, \quad -5 \leq x \leq 5.$$

   (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n+1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?

(b) Repeat part (a) but now use the interpolation points

$$x_j = 5\cos\frac{(2j-1)\pi}{2n+2}, \quad j = 1, \ldots, n+1.$$

What difference do you observe ?

7. Ex.1, p.156, *Atkinson & Han.*
8. Ex.5, p.156, *Atkinson & Han.*
9. Consider the function s(x) defined as

$$s(x) = \begin{cases} 28 + 25x + 9x^2 + x^3, & -3 \le x \le -1, \\ 26 + 19x + 3x^2 - x^3, & -1 \le x \le 0, \\ 26 + 19x + 3x^2 - 2x^3, & 0 \le x \le 3, \\ -163 + 208x - 60x^2 + 5x^3, & 3 \le x \le 4. \end{cases}$$

Show that $s(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. Be sure to state explicitly each of the properties of $s(x)$ which are necessary for this to be true.