AMSC/CMSC 466 Dr. Wolfe ASSIGNMENT #4 Due November 10, 2011

1. The vapor pressure P of water (in bars) as a function of temperature $T\,(^{\circ}C)$ is

T	0	10	20	30
P(T)	.006107	.012277	.023378	.042433
T	40	50	60	70
P(T)	.073774	.12338	.19924	.31166
T	80	90	100	110
P(T)	.47364	.70112	1.01325	1.22341

Find and plot the polynomials of degree 1, 2 and 3 which best fit this data in the sense of least squares. The MATLAB functions POLYVAL and POLYFIT give you exactly what you need.

2. For the data of problem 1, find the cubic polynomial $p_3(x) = p_0 + p_1x + p_2x^2 + p_3x^3$ interpolating the data in the sense of least squares by constructing the 12×4 data matrix A and finding the vector $(p_0, p_1, p_2, p_3)^T$ in four different ways:
   (a) By using the backslash operator.
   (b) By forming and solving the normal equations. Note the condition number of the matrix A^TA.
   (c) By using the QR decomposition.
   (d) By using the Singular-Value Decomposition.
   All of this is quite easy in MATLAB. Compare with the values found in problem 1.

3. Write a MATLAB program to evaluate $I = \int_a^b f(x)\,dx$ using the trapezoidal rule with n subdivisions, calling the result I_n. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

$$\text{(a)} \quad \int_0^1 \sqrt{9+x^2}\,dx \qquad \text{(b)} \quad \int_0^1 x^{1/4}\,dx$$

The exact value of the integral in (a) is 3.05466450615185.
Analyze emperically the rate of convergence of I_n to I by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \text{ and } p_n = \frac{\log(R_n)}{\log(2)}$$

In part (b) compute the extrapolated approximation to I,

$$I \tilde{=} I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

for $n = 128$.

4. Repeat problem 3 using Simpson's rule.

5. Apply the corrected trapezoidal rule to the integral in problem 3(a). Compare the results with those of problem 4 for Simpson's rule.

6. Find approximate values of the integrals in problem 3 by computing the Romberg integral $I_{32}^{(5)}$ where $I_n^{(0)}$ is the n-panel trapezoid approximation and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_{n/2}^{(k-1)}}{4^k - 1}$$

for n divisible by 2^k.

7. The 10 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x)\, dx \approx \sum_{i=0}^{9} w_i f(\frac{i}{9})$$

with the w_i determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots x^9$.
(a) Use MATLAB to find the weights w_i.
(b) Apply the rule to the integrals in 3(a) and 3(b). Note the errors.

8. Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at $x = 0, h, 2h$. Use this to derive a numerical integration formula I_h for $I = \int_0^{3h} f(x)\, dx$. Use a Taylor series expansion of $f(x)$ to show

$$I - I_h = \frac{3}{8} h^4 f^{'''}(0) + O(h^5).$$