Homework #5 (due May 1, 2012)

1. Let $\psi(x; \theta)$ be an estimating vector function for a family $\mathcal{P} = \{P_\theta, \theta \in \Theta \subset \mathbb{R}^s\}$ of distributions of $X \in (\mathcal{X}, \mathcal{A})$.

Show that if $T$ is sufficient for $\mathcal{P}$, the estimating vector function $\hat{\psi}(T; \theta) = E_\theta(\psi(X; \theta)|T)$ is better than $\psi$, i.e., $I_{\hat{\psi}} \geq I_\psi$.

2. Let $X_1 \sim P^{(1)}_\theta$, $X_2 \sim P^{(2)}_\theta$ be independent random elements, and $\psi_1(x_1 : \theta), \psi_2(x_2 : \theta)$ be estimating functions for $\mathcal{P}^{(1)}$, $\mathcal{P}^{(2)}$, respectively.

(i) Show that $\psi(x_1, x_2; \theta) = \psi_1(x_1; \theta) + \psi_2(x_2; \theta)$ or $\psi(x_1, x_2; \theta) = \psi_1(x_1; \theta) - \psi_2(x_2; \theta)$ is an estimating function for the family of distributions of $(X, Y)$.

(ii) Show that in general $I_\psi \neq I_{\psi_1} + I_{\psi_2}$ but for the Fisher estimating functions the equality sign holds.