1. Let $A$ be the $3 \times 4$ matrix $A = \begin{bmatrix} 0 & 2 & 8 & -1 \\ 1 & 0 & 3 & 2 \\ 1 & 1 & 7 & 0 \end{bmatrix}$.

   (a) (Fifteen points) Use row operations to determine the reduced row echelon form which is row-equivalent to $A$.

   (b) (Five points) Do the columns of $A$ span $\mathbb{R}^3$? Briefly explain why or why not.
2. (a) (Five points) What is the definition of the span of a set of vectors \( \{ \vec{v}_1, \ldots, \vec{v}_n \} \)?

(b) (Fifteen points) Is the vector \(
\begin{bmatrix}
5 \\
-6 \\
2
\end{bmatrix}
\) in the span of \(
\begin{bmatrix}
1 \\
4 \\
0
\end{bmatrix}
\) and \(
\begin{bmatrix}
-3 \\
1 \\
0
\end{bmatrix}
\) ? Explain why or why not.
3. Suppose $A$ is the following $4 \times 4$ matrix. I’ve helpfully computed $\text{rref}(A)$ for you. (Thank me later.)

$$\begin{pmatrix}
-4 & -9 & 10 & -3 \\
-6 & -8 & 4 & -3 \\
2 & 3 & -2 & 1 \\
-10 & -17 & 14 & -6
\end{pmatrix}, \quad \begin{pmatrix}
1 & 0 & 2 & 0 \\
0 & 1 & -2 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

(a) (Six points) Solve the matrix equation $A\vec{x} = \vec{0}$. If there are infinitely many solutions, give a set of vectors that spans the solution set.

(b) (Six points) Are the columns of $A$ linearly independent? If not, tell me a nontrivial linear dependence relation they satisfy.

(c) (Eight points) Consider the function $\vec{x} \mapsto A\vec{x}$. This is a function from $\mathbb{R}^4$ to $\mathbb{R}^4$. It is neither one-to-one nor onto. Explain why each of these properties fail to hold for this function.
4. (Five points each) Each part of this problem is a question whose answer is “yes, always”, “sometimes yes but also sometimes no”, and “no, never”. Circle the correct choice of the three. No work is required for these problems: I’ll give full credit if you circle the correct choice. If you want to explain yourself with examples and/or counterexamples, they may be worth partial credit if your answer is incorrect.

(a) Suppose $M$ is a $5 \times 9$ matrix. Is $\vec{x} \mapsto M\vec{x}$ a function from $\mathbb{R}^5 \to \mathbb{R}^9$?

- Yes, always
- Sometimes yes but sometimes no
- No, never

(b) Suppose $\text{rref}(A)$ has a zero row at the bottom. Does the equation $A\vec{x} = \vec{0}$ have infinitely many solutions?

- Yes, always
- Sometimes yes but sometimes no
- No, never

(c) Suppose that $M$ and $N$ are two different matrices and that $\text{rref}(M) = \text{rref}(N)$. Is there a sequence of row operations that transforms $M$ to $N$?

- Yes, always
- Sometimes yes but sometimes no
- No, never

(d) Suppose $T$ is a linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^3$. Is $T$ surjective?

- Yes, always
- Sometimes yes but sometimes no
- No, never
5. (a) (Ten points) Write down the standard matrix for the transformation \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) which consists of the following operations: scaling the \( x \) direction by a factor of 3, scaling the \( y \) direction by a factor of 4, and then performing a rotation of 120 degrees counterclockwise.

(b) (Ten points) Suppose that the set \( \{ \vec{u}, \vec{v}, \vec{w} \} \) of three vectors in \( \mathbb{R}^3 \) is linearly dependent. Let \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) be a linear transformation. Explain why the set \( \{ T(\vec{u}), T(\vec{v}), T(\vec{w}) \} \) is also linearly dependent.

Please copy the honor pledge below and sign your name next to it.

“I pledge on my honor that I have not given or received any unauthorized assistance on this examination.”