Write legibly and show all work. No partial credit can be given for an unjustified, incorrect answer. Put your name in the top right corner and sign the honor pledge at the end of the exam. If you need more room than what’s given, please continue onto the back.

1. (a) (Five points) Let \( A = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ 3 & -2 & -1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix} \).

Calculate the product \( AB \).

(b) (Fifteen points) Find the inverse of the matrix \( A \) from the previous part.
2. (a) (Ten points) Consider the matrix

\[ A = \begin{bmatrix}
-1 & 1 & 1 \\
1 & x & -1 \\
2 & 1 & x
\end{bmatrix}, \]

where \( x \) is a variable. Calculate \( \det(A) \). (Your answer could have an \( x \) in it.)

(b) (Five points) List all values of \( x \) that make the matrix singular.

(c) (Five points) Plug in for \( x \) one of the values you got from the previous part to get a \( 3 \times 3 \) matrix with numbers as the entries. Find a linear combination among the columns of that matrix. [Hint. One of the values will be easier than the other.]
3. Below you’ll find the \( LU \)-factorization of a \( 3 \times 3 \) matrix \( B \).

\[
B = \begin{bmatrix} -3 & 0 & -1 \\ 3 & 1 & 1 \\ 15 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}.
\]

(a) (Fifteen points) Use the factorization to solve the equation \( B\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -11 \end{bmatrix} \).

(b) (Five points) Determine \( \det(B) \).
4. (Five points each) It’s that time again! Each part of this problem is a question whose answer is “yes, always”, “sometimes yes but also sometimes no”, and “no, never”. Circle the correct choice of the three. No work is required for these problems: I’ll give full credit if you circle the correct choice. If you want to explain yourself with examples and/or counterexamples, they may be worth partial credit if your answer is incorrect.

(a) Let $A$ be an invertible $n \times n$ matrix. Is $\text{rref}(A) = I_n$?

<table>
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<tr>
<th>Yes, always</th>
<th>Sometimes yes but sometimes no</th>
<th>No, never</th>
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(b) Suppose $A$ is a square matrix and that $\det A^4 = 0$. Is the function $\vec{x} \mapsto A\vec{x}$ surjective?

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<th>Yes, always</th>
<th>Sometimes yes but sometimes no</th>
<th>No, never</th>
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(c) Let $B$ be an invertible $2 \times 2$ matrix. Is $\det(3B) = 3\det(B)$?

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<th>Yes, always</th>
<th>Sometimes yes but sometimes no</th>
<th>No, never</th>
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(d) Suppose that $A$ and $B$ are invertible matrices. Is $(A + B)^2 = A^2 + 2AB + B^2$?

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<th>Yes, always</th>
<th>Sometimes yes but sometimes no</th>
<th>No, never</th>
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5. (Six points each) In both parts of this problem I’ve given you two matrices, $M$ and $N$, that differ by a row operation. Write down an elementary matrix $E$ which satisfies $EM = N$.

(a) $M = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 0 & 1 \\ -14 & 0 & 2 \end{bmatrix}$, $N = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 0 & 1 \\ 1 & 0 & -\frac{1}{7} \end{bmatrix}$

(b) $M = \begin{bmatrix} 1 & 2 & -1 & -1 \\ -2 & 4 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 8 & -3 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

6. (Eight points) Let $A$ be a $3 \times 2$ matrix and $B$ be a $2 \times 3$ matrix. Explain why $AB$ cannot equal $I_3$. [Hint. Explain first why you can solve the equation $B\vec{x} = \vec{0}$.]

Please copy the honor pledge below and sign your name next to it.

“I pledge on my honor that I have not given or received any unauthorized assistance on this examination.”