Solution of Quiz 3

12PM

Problem 1

$$\begin{aligned}\int \sin^2(x)\,dx &= \int \frac{1-\cos(2x)}{2}\,dx \\ &= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(2x)\,dx \\ &= \frac{1}{2}x - \frac{\sin(2x)}{4} + C\end{aligned}$$

Problem 2

$$\begin{aligned}\int x\cos(x^2)\,dx &= \frac{1}{2}\int \cos(u)\,du \quad (u = x^2,\ du = 2xdx) \\ &= \frac{\sin(u)}{2} + C \\ &= \frac{\sin(x^2)}{2} + C\end{aligned}$$

Problem 3

$$\begin{aligned}\int x^2\sin(x)\,dx &= -x^2\cos x + 2\int x\cos x\,dx \quad u = x^2, du = 2xdx, dv = \sin x dx, v = -\cos x \\ &= -x^2\cos x + 2x\sin x - \int 2\sin x dx \quad \text{(using integration by parts again)} \\ &= -x^2\cos x + 2x\sin x + 2\cos x + C\end{aligned}$$

Problem 4

$$\begin{aligned}\int \sin^3 x\cos^2 x dx &= \int \sin(\sin^2 x\cos^2 x)dx \\ &= \int \sin x((1-\cos^2 x)\cos^2 x)dx \\ &= -\int (1-u^2)u^2 du \quad \text{where } u = \cos x,\ du = -\sin x dx \\ &= \int u^4 - u^2 du \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C \\ &= \frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + C\end{aligned}$$

1PM

Problem 1

$$\begin{aligned}\int \sin^2(x)\,dx &= \int \frac{1-\cos(2x)}{2}\,dx \\ &= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(2x)\,dx \\ &= \frac{1}{2}\,x - \frac{\sin(2x)}{4} + C\end{aligned}$$

Problem 2

$$\begin{aligned}\int x^2 \sin(x^3)\,dx &= \frac{1}{3}\int \sin(u)\,du \quad \text{where } u = x^3,\ du = 3x^2dx \\ &= -\frac{\cos(u)}{3} + C \\ &= -\frac{\cos(x^3)}{3} + C\end{aligned}$$

Problem 3

$$\begin{aligned}\int \sin(x)e^x\,dx &= e^x \sin x - \int e^x \cos x\,dx \quad u = \sin x, du = \cos x dx, dv = e^x dx, v = e^x \\ &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \quad \text{(using integration by parts again)}\end{aligned}$$

Now, adding $\int \sin(x)e^x$ to both sides,

$$\begin{aligned}2\int \sin(x)e^x dx &= e^x \sin(x) - e^x \cos(x) \\ \text{So, } \int \sin(x)e^x dx &= \frac{e^x}{2}(\sin(x) - \cos(x)) + C\end{aligned}$$

Problem 4

$$\begin{aligned}\int \sin(x)\cos^4(x)dx &= -\int u^4 du \quad \text{where } u = \cos x,\ du = -\sin x dx \\ &= -\frac{u^5}{5} + C \\ &= -\frac{\cos^5(x)}{5} + C\end{aligned}$$

2PM

Problem 1

$$\begin{aligned} \int \cos^2(x)\,dx &= \int \frac{1+\cos(2x)}{2}\,dx \\ &= \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\,dx \\ &= \frac{1}{2}x + \frac{\sin(2x)}{4} + C \end{aligned}$$

Problem 2

$$\begin{aligned} \int x^2\cos(x^3)\,dx &= \frac{1}{3}\int \cos(u)\,du \quad \text{where } u = x^3,\ du = 3x^2dx \\ &= \frac{\sin(u)}{3} + C \\ &= \frac{\sin(x^3)}{3} + C \end{aligned}$$

Problem 3

$$\begin{aligned} \int e^x\cos(x)\,dx &= e^x\cos x + \int e^x\sin x\,dx \quad u = \cos x, du = -\sin x dx, dv = e^x dx, v = e^x \\ &= e^x\cos x + e^x\sin x - \int e^x\cos x dx \quad \text{(using integration by parts again)} \end{aligned}$$

Now, adding $\int e^x\cos(x)$ to both sides,

$$\begin{aligned} 2\int e^x\cos(x)dx &= e^x\cos(x) + e^x\sin(x) \\ \text{So, } \int e^x\cos(x)dx &= \frac{e^x}{2}(\cos(x)+\sin(x)) + C \end{aligned}$$

Problem 4

$$\begin{aligned} \int \sin^4(x)\cos(x)dx &= \int u^4du \quad \text{where } u = \sin x,\ du = \cos x dx \\ &= \frac{u^5}{5} + C \\ &= \frac{\sin^5(x)}{5} + C \end{aligned}$$

3PM

Problem 1

$$\begin{aligned}\int \sin^2(x)\,dx &= \int \frac{1-\cos(2x)}{2}\,dx \\ &= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(2x)\,dx \\ &= \frac{1}{2}\,x - \frac{\sin(2x)}{4} + C\end{aligned}$$

Problem 2

$$\begin{aligned}\int x\sin(x^2)\,dx &= \frac{1}{2}\int \sin(u)\,du \quad (u = x^2,\ du = 2xdx) \\ &= -\frac{\cos(u)}{2} + C \\ &= -\frac{\cos(x^2)}{2} + C\end{aligned}$$

Problem 3

$$\begin{aligned}\int x^2\ln(x)\,dx &= \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx \quad u = \ln x, du = \frac{dx}{x}, dv = x^2dx, v = \frac{x^3}{3} \\ &= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C\end{aligned}$$

Problem 4

$$\begin{aligned}\int \sin^2 x\cos^2 x dx &= \int (\sin x\cos x)^2 dx \\ &= \int (\frac{1}{2}\sin(2x))^2 dx \\ &= \frac{1}{4}\int \sin^2(2x)dx \\ &= \frac{1}{8}\int \sin^2(u)du \quad (u = 2x) \\ &= \frac{1}{8}(\frac{u}{2} - \frac{\sin(2u)}{4}) + C \quad \text{(Using Problem 1)} \\ &= \frac{1}{8}(x - \frac{\sin(4x)}{4}) + C\end{aligned}$$