

MATH 141, FALL 2010, Midterm 3, sample

1) Determine if the following improper integral is convergent or divergent. If convergent, find its value.

$$\int_{e^2}^{\infty} \frac{1}{x(\ln(x))^2} dx.$$

2) Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{(-1)^k 4^{k-1}}{3^{(2k)}}.$$

3) Find the radius of convergence of the following power series

$$\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2 \cdot 2 \cdot 6 \cdots (2n)} x^n.$$

4) Evaluate

$$\lim_{n \rightarrow \infty} (e^{2n} - 1)^{1/n}.$$

5) Give an explicit numerical bound on the error in approximating  $\sin(1)$  by the first two terms in the Taylor series

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$