

MATH 410, HW 2

1. Let A and B be compact subsets of \mathbb{R} . Show that their union $A \cup B$ is also compact.
2. Let A and B be sets in \mathbb{R} . If the union $A \cup B$ is a compact set, is it true that both A and B must also be compact? Justify your answer.
3. Using the ϵ - δ “language” state formally the following: a function f defined on an interval (a, b) , containing a point x_0 , is NOT continuous at x_0 .
4. Find an example of a continuous function f defined on \mathbb{R} such that its range of values equals to the open interval $(0, 1)$.
5. Let a function $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous and periodic (i.e., there exists $T \in \mathbb{R}$ such that $f(x + T) = f(x)$, for all $x \in \mathbb{R}$). Show that such f is necessarily uniformly continuous.