MATH 410, HW 4

1. Find the indefinite integral:
\[ \int \frac{dx}{\sqrt{(x - a)(x - b)}}. \]

2. Find the indefinite integral:
\[ \int \sqrt{\frac{a + x}{a - x}} \, dx. \]

3. Without using the properties of logarithm, prove that for all \( x > 0 \) and for any rational number \( a > 0 \):
\[ \int_1^x \frac{dt}{t} = a \int_1^x \frac{dt}{t}. \]

4. Suppose \( f : [a, b] \rightarrow \mathbb{R} \) is integrable. Prove that
\[ \left| \int_a^x f(t) \, dt - \int_a^y f(t) \, dt \right| \leq C|x - y|, \]
for some constant \( C > 0 \).

5. Suppose that functions \( f, g : [a, b] \rightarrow \mathbb{R} \) are continuous on \([a, b]\). Prove that
\[ \int_a^b |f + g| \leq \int_a^b |f| + \int_a^b |g|. \]