1. Write explicitly the $4 \times 4$ matrix of the Discrete Haar transform. Apply this matrix to vector $(1, 0, 1, 0)$.

2. Let \( \{ h(k) : k = 0, \ldots, L \} \), \( \{ g(k) : k = 0, \ldots, L \} \) be a pair of Conjugate Quadrature Mirror Filters of finite length \( L + 1 < \infty \). Let \( c \in \mathbb{R}^d \) \( (d > L, \text{ d even}) \). Let \( H(c)(n) = \sum_k h(k) c(k + 2n) \) and \( G(c)(n) = \sum_k g(k) c(k + 2n) \). Thus, \( H \) and \( G \) can be identified with \((d/2) \times d\) matrices. Let \( A^* \) denote the adjoint to \( A \). Show that \( HH^* = GG^* = Id \).

3. Let \( \{ h(k) : k = 0, \ldots, L \} \), \( \{ g(k) : k = 0, \ldots, L \} \) be a pair of Conjugate Quadrature Mirror Filters of finite length \( L + 1 < \infty \). Let \( c \in \mathbb{R}^d \) \( (d > L, \text{ d even}) \). Let \( H(c)(n) = \sum_k h(k) c(k + 2n) \) and \( G(c)(n) = \sum_k g(k) c(k + 2n) \). Thus, \( H \) and \( G \) can be identified with \((d/2) \times d\) matrices. Let \( A^* \) denote the adjoint to \( A \). Show that \( HG^* = GH^* = 0 \).

4. Show that if \( M \) is an arbitrary integer and if \( \{ h(k) : k = 0, \ldots, L \} \) is a Quadrature Mirror Filter, then so is the sequence 
\[
g(k) = (-1)^k h(2M - 1 - k).
\]

5. Prove that the \( N \times N \) matrix of the Discrete Haar Transform is unitary.

6. Find the Lagrange polynomial through the points \((1, 1), (2; 2), (3; 3)\).

7. Find the expansion in Chebyshev polynomials \( T_0(x), T_1(x), T_2(x) \) of the function \( f(x) = 1 + x^2 \) dened for \( x \in [-1, 1] \).

8. Suppose that \( f(x) = c \) is a constant function. Show that for any sampling of \( f \), the piecewise linear approximation exactly equals \( f \).