MATH 416, Spring 2010, Midterm 3 - Review

1. Write explicitly the 4×4 matrix of the Discrete Haar transform. Apply this matrix to vector $(1, 0, 1, 0)$.

2. Let $\{h(k) : k = 0, \ldots, L\}$, $\{g(k) : k = 0, \ldots, L\}$ be a pair of Conjugate Quadrature Mirror Filters of nite length $L + 1 < \infty$. Let $c \in \mathbb{R}^d$ ($d > L$, d even) . Let $H(c)(n) = \sum_k h(k)c(k + 2n)$ and $G(c)(n) = \sum_k g(k)c(k + 2n)$. Thus, H and G can be identified with $(d/2) \times d$ matrices. Let A^* denote the adjoint to A. Show that $HH^* = GG^* = Id$.

3. Let $\{h(k) : k = 0, \ldots, L\}$, $\{g(k) : k = 0, \ldots, L\}$ be a pair of Conjugate Quadrature Mirror Filters of nite length $L + 1 < \infty$. Let $c \in \mathbb{R}^d$ ($d > L$, d even) . Let $H(c)(n) = \sum_k h(k)c(k + 2n)$ and $G(c)(n) = \sum_k g(k)c(k + 2n)$. Thus, H and G can be identified with $(d/2) \times d$ matrices. Let A^* denote the adjoint to A. Show that $HG^* = GH^* = 0$.

4. Show that if M is an arbitrary integer and if $\{h(k) : k = 0, \ldots, L\}$ is a Quadrature Mirror Filter, then so is the sequence

$$g(k) = (-1)^k \overline{h(2M - 1 - k)}.$$

5. Prove that the $N \times N$ matrix of the Discrete Haar Transform is unitary.

6. Find the Lagrange polynomial through the points (1, 1), (2; 2), (3; 3).

7. Find the expansion in Chebyshev polynomials $T_0(x), T_1(x), T_2(x)$ of the function $f(x) = 1 + x^2$ dened for $x \in [-1, 1]$.

8. Suppose that $f(x) = c$ is a constant function. Show that for any sampling of f, the piecewise linear approximation exactly equals f.