8 AM

Question 1

Use the disk method. Then

\[ V = \int_{0}^{\pi/2} \pi (\sqrt{\cos(x)})^2 dx \]
\[ = \int_{0}^{\pi/2} \pi \cos(x) dx \]
\[ = \pi \sin(x)|_{0}^{\pi/2} \]
\[ = \pi \]

Question 2

\[ L = \int_{1}^{5} \sqrt{1 + f'(x)^2} dx \]
\[ = \int_{1}^{5} \sqrt{5} dx \]
\[ = 4\sqrt{5} \]

Question 3

Use the shell method

\[ V = \int_{2}^{3} 2\pi x (f(x) - g(x)) dx \]
\[ = \int_{2}^{3} 2\pi x (1 - (x - 2)) dx \]
\[ = \int_{2}^{3} 6\pi x - 2\pi x^2 dx \]
\[ = 3\pi x^2 - \frac{2}{3} \pi x^3|_{2}^{3} \]
\[ = \frac{7\pi}{3} \]
Question 4

\[ L = \int_{0}^{1} \sqrt{1 + f'(x)^2} \, dx \]
\[ = \int_{0}^{1} \sqrt{1 + (2x^{1/2})^2} \, dx \]
\[ = \int_{0}^{1} \sqrt{1 + 4x} \, dx \]
\[ = \frac{1}{4} \int \sqrt{u} \, du \] (Using u-substitution)
\[ = \frac{1}{6} (1 + 4x)^{3/2} \bigg|_{0}^{1} \]
\[ = \frac{1}{6} (5^{3/2} - 1) \]

9 AM

Question 1

Use the disk method. Then

\[ V = \int_{0}^{\pi/2} \pi (\sqrt{\cos(x)})^2 \, dx \]
\[ = \int_{0}^{\pi/2} \pi \cos(x) \, dx \]
\[ = \pi \sin(x) \bigg|_{0}^{\pi/2} \]
\[ = \pi \]

Question 2

\[ L = \int_{1}^{5} \sqrt{1 + f'(x)^2} \, dx \]
\[ = \int_{1}^{5} \sqrt{10} \, dx \]
\[ = 4\sqrt{10} \]
Question 3

Use the shell method

\[ V = \int_{1}^{3} 2\pi x (f(x) - g(x)) \, dx \]

\[ = \int_{1}^{3} 2\pi x (2x - 1) \, dx \]

\[ = \int_{1}^{3} 4\pi x^2 - 2\pi x \, dx \]

\[ = \frac{4}{3} \pi x^3 - \pi x^2 \bigg|_{1}^{3} \]

\[ = \frac{80}{3} \pi \]

Question 4

A sphere can be formed by rotating the line \( f(x) = \sqrt{r^2 - x^2} \) about the x-axis from \(-r\) to \(r\). Then

\[ V = \int_{-r}^{r} \pi (\sqrt{r^2 - x^2})^2 \, dx \]

\[ = \int_{-r}^{r} \pi r^2 - \pi x^2 \, dx \]

\[ = \pi r^2 x - \frac{\pi}{3} x^3 \bigg|_{-r}^{r} \]

\[ = \left( \pi r^3 - \frac{\pi}{3} r^3 \right) - \left( -\pi r^3 - \frac{\pi}{3} (-r)^3 \right) \]

\[ = \frac{4}{3} \pi r^3 \]

10 AM

Question 1

First note that \( \cos(x) < 0 \) for \( x \in \left( \frac{\pi}{2}, \pi \right) \), and so \( \sqrt{\cos(x)} \) is undefined there. We will instead compute the volume on the interval on \([0, \frac{\pi}{2}]\). Use the disk method. Then

\[ V = \int_{0}^{\pi/2} \pi (\sqrt{\cos(x)})^2 \, dx \]

\[ = \int_{0}^{\pi/2} \pi \cos(x) \, dx \]

\[ = \pi \sin(x) \bigg|_{0}^{\pi/2} \]

\[ = \pi \]
Question 2

\[ L = \int_1^5 \sqrt{1 + f'(x)^2} \, dx \]
\[ = \int_1^5 \sqrt{2} \, dx \]
\[ = 4\sqrt{2} \]

Question 3

Use the shell method

\[ V = \int_1^2 2\pi x (f(x) - g(x)) \, dx \]
\[ = \int_1^2 2\pi x (3x - (x - 1)) \, dx \]
\[ = \int_1^2 4\pi x^2 + 2\pi x \, dx \]
\[ = \frac{4}{3} \pi x^3 + \pi x^2 \bigg|_1 \]
\[ = \frac{37}{3} \pi \]

Question 4

A sphere can be formed by rotating the line \( f(x) = \sqrt{r^2 - x^2} \) about the x-axis from \(-r\) to \(r\). Then

\[ V = \int_{-r}^{r} \pi (\sqrt{r^2 - x^2})^2 \, dx \]
\[ = \int_{-r}^{r} \pi r^2 - \pi x^2 \, dx \]
\[ = \pi r^2 x - \frac{\pi}{3} x^3 \bigg|_{-r}^{r} \]
\[ = (\pi r^3 - \frac{\pi}{3} r^3) - (\pi r^3 - \frac{\pi}{3} (r)^3) \]
\[ = \frac{4}{3} \pi r^3 \]
11 AM

**Question 1**

Use the disk method. Then

\[
V = \int_{0}^{\pi/4} \pi (\sqrt{\sin(x)})^2 \, dx
\]

\[
= \int_{0}^{\pi/4} \pi \sin(x) \, dx
\]

\[
= -\pi \cos(x) \bigg|_{0}^{\pi/4}
\]

\[
= 2 - \sqrt{2}
\]

\[
= \frac{2 - \sqrt{2}}{2} \pi
\]

**Question 2**

\[
L = \int_{1}^{5} \sqrt{1 + f'(x)^2} \, dx
\]

\[
= \int_{1}^{5} \sqrt{2} \, dx
\]

\[
= 4\sqrt{2}
\]

**Question 3**

Use the shell method

\[
V = \int_{1}^{3} 2\pi x (f(x) - g(x)) \, dx
\]

\[
= \int_{0}^{3} 2\pi x (2x - x) \, dx
\]

\[
= \int_{0}^{3} 2\pi x^2 \, dx
\]

\[
= \frac{2}{3} \pi x^3 \bigg|_{0}^{3}
\]

\[
= \frac{2}{3} \pi (3)^3
\]

\[
= 18\pi
\]
Question 4

\[
L = \int_1^2 \sqrt{1 + f'(x)^2} \, dx \\
= \int_1^2 \sqrt{1 + (3x^{1/2})^2} \, dx \\
= \int_1^2 \sqrt{1 + 9x} \, dx \\
= \frac{1}{9} \int \sqrt{u} \, du \quad \text{(Using u-substitution)} \\
= \frac{2}{27} (1 + 9x)^{3/2} \bigg|_1^2 \\
= \frac{2}{27} (19^{3/2} - 10^{3/2}) 
\]

12 PM

Question 1

First note that \( \cos(x) < 0 \) for \( x \in (\pi, \frac{3\pi}{2}) \), and so \( \sqrt{\cos(x)} \) is undefined there. We will instead use the function \( f(x) = \sqrt{|\cos(x)|} \). Use the disk method. Then

\[
V = \int_{\pi}^{3\pi/2} \pi (\sqrt{|\cos(x)|})^2 \, dx \\
= \int_{\pi}^{3\pi/2} \pi |\cos(x)| \, dx \\
= \int_{\pi}^{3\pi/2} \pi (-\cos(x)) \, dx \\
= -\pi \sin(x) \bigg|_{\pi}^{3\pi/2} \\
= \pi 
\]

Question 2

\[
L = \int_2^4 \sqrt{1 + f'(x)^2} \, dx \\
= \int_2^4 \sqrt{2} \, dx \\
= 2\sqrt{2} 
\]
Question 3
Use the shell method

\[
V = \int_2^3 2\pi x (f(x) - g(x))\,dx \\
= \int_2^3 2\pi x (1 - (x - 2))\,dx \\
= \int_2^3 6\pi x - 2\pi x^2\,dx \\
= 3\pi x^2 - \frac{2}{3}\pi x^3|_2^3 \\
= \frac{7\pi}{3}
\]

Question 4
A sphere can be formed by rotating the line \( f(x) = \sqrt{r^2 - x^2} \) about the x-axis from \(-r\) to \(r\). Then

\[
V = \int_{-r}^r \pi (\sqrt{r^2 - x^2})^2\,dx \\
= \int_{-r}^r \pi r^2 - \pi x^2\,dx \\
= \pi r^2 x - \frac{\pi}{3} x^3|_{-r}^r \\
= (\pi r^3 - \frac{\pi}{3} r^3) - (-\pi r^3 - \frac{\pi}{3} (-r)^3) \\
= \frac{4}{3}\pi r^3
\]

1 PM

Question 1
First note that \(\sin(x) < 0\) for \(x \in (-\pi, 0)\), and so \(\sqrt{\sin(x)}\) is undefined there. We will instead use the function \(f(x) = \sqrt{|\sin(x)|}\). Use the disk method. Then

\[
V = \int_{-\pi}^0 \pi (\sqrt{|\sin(x)|})^2\,dx \\
= \int_{-\pi}^0 \pi |\sin(x)|\,dx \\
= \int_{-\pi}^0 \pi (-\sin(x))\,dx \\
= \pi \cos(x)|_{-\pi}^0 \\
= 2\pi
\]
Question 2

\[ L = \int_1^5 \sqrt{1 + f'(x)^2} \, dx \]
\[ = \int_1^5 \sqrt{10} \, dx \]
\[ = 4\sqrt{10} \]

Question 3

Use the shell method

\[ V = \int_1^3 2\pi x (f(x) - g(x)) \, dx \]
\[ = \int_1^3 2\pi x (3 - (-2)) \, dx \]
\[ = \int_1^3 10\pi x \, dx \]
\[ = 5\pi x^2 \bigg|_1^3 \]
\[ = 40\pi \]

Question 4

\[ L = \int_0^2 \sqrt{1 + f'(x)^2} \, dx \]
\[ = \int_0^2 \sqrt{1 + \left(\frac{3}{2}x^{1/2}\right)^2} \, dx \]
\[ = \int_0^2 \sqrt{1 + \frac{9}{4}x} \, dx \]
\[ = \frac{4}{9} \int \sqrt{u} \, du \text{ (Using u-substitution)} \]
\[ = \frac{8}{27} u^{3/2} \bigg|_* \]
\[ = \frac{8}{27} (1 + \frac{9}{4}x^{3/2} \bigg|_0 \]
\[ = \frac{8}{27} ((\frac{11}{2})^{3/2} - 1) \]