8 AM

Question 1

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$, so

$$\begin{aligned}
\sec\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) &= \sec\left(\frac{\pi}{3}\right) \\
&= \frac{1}{\cos\left(\frac{\pi}{3}\right)} \\
&= \frac{1}{1/2} \\
&= 2
\end{aligned}$$

Question 2

Since the limit as $x \to 0^+$ is zero in both the numerator and the denominator, we use L'Hopital's rule:

$$\begin{aligned}
\lim_{x\to 0^+} \frac{1-\cos(x)}{\sin(x)} &= \lim_{x\to 0^+} \frac{\sin(x)}{\cos(x)} \\
&= \frac{\sin(0)}{\cos(0)} \\
&= \frac{0}{1} \\
&= 0
\end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$\begin{aligned}
x &= -4y^3 - 1 \\
x + 1 &= -4y^3 \\
-\frac{x+1}{4} &= y^3 \\
\left(-\frac{x+1}{4}\right)^{1/3} &= y = f^{-1}(x)
\end{aligned}$$

Question 4

We proceed by substitution, letting $u = ex$ and $du = edx$:

$$\begin{aligned} \int e^{ex} dx &= \int e^u \frac{1}{e} du \\ &= \frac{1}{e} e^u + C \\ &= e^{-1} e^{ex} + C \\ &= e^{ex-1} + C \end{aligned}$$

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Question 1

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$, so

$$\begin{aligned} \sec\left(\sin^{-1}\left(\frac{1}{2}\right)\right) &= \sec\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\cos\left(\frac{\pi}{6}\right)} \\ &= \frac{1}{\sqrt{3}/2} \\ &= \frac{2}{\sqrt{3}} \end{aligned}$$

Question 2

Since the limit as $x \to 0^+$ is zero in both the numerator and the denominator, we use L'Hopital's rule:

$$\begin{aligned} \lim_{x \to 0^+} \frac{1 - \cos(x)}{\sin(x)} &= \lim_{x \to 0^+} \frac{\sin(x)}{\cos(x)} \\ &= \frac{\sin(0)}{\cos(0)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$\begin{aligned} x &= y^2 - 1 \\ x + 1 &= y^2 \\ \sqrt{x+1} &= |y| \\ \sqrt{x+1} &= y = f^{-1}(x) \end{aligned}$$

The choice of positive square root is made to give us the inverse on $[0, \infty)$.

Question 4

We proceed by substitution, letting $u = 2x$ and $du = 2dx$:

$$\begin{aligned} \int 2^{2x} dx &= \int 2^u \frac{1}{2} du \\ &= \frac{1}{2} \int e^{\ln 2^u} du \\ &= \frac{1}{2} \int e^{u \ln 2} du \end{aligned}$$

Now letting $v = \ln(2)u$ and $dv = \ln(2)du$:

$$\begin{aligned} \frac{1}{2} \int e^{u \ln 2} du &= \frac{1}{2} \int e^v \frac{1}{\ln(2)} dv \\ &= \frac{1}{2\ln(2)} e^v + C \\ &= \frac{1}{2\ln(2)} e^{\ln(2)u} + C \\ &= \frac{1}{2\ln(2)} e^{\ln(2)2x} + C \\ &= \frac{1}{2\ln(2)} 2^{2x} + C \end{aligned}$$

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Question 1

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$, so

$$\begin{aligned} \csc\left(\sin^{-1}\left(\frac{1}{2}\right)\right) &= \csc\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\sin\left(\frac{\pi}{6}\right)} \\ &= \frac{1}{1/2} \\ &= 2 \end{aligned}$$

Question 2

Since the limit as $x \to 0^+$ is zero in both the numerator and the denominator, we use L'Hopital's rule:

$$\begin{aligned} \lim_{x \to 0^+} \frac{1 - \cos(x)}{\sin(x)} &= \lim_{x \to 0^+} \frac{\sin(x)}{\cos(x)} \\ &= \frac{\sin(0)}{\cos(0)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$\begin{aligned} x &= y^3 - 1 \\ x + 1 &= y^3 \\ \sqrt[3]{x+1} &= y = f^{-1}(x) \end{aligned}$$

Question 4

We know that:

$$\int \frac{dx}{a^2 + u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

So by setting $u = x$ and $a = \sqrt{2}$, we obtain:

$$\begin{aligned} \int 2dx2 + x^2 &= 2 \int dxa^2 + u^2 \\ &= 2\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{a}\right) \\ &= 2\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \end{aligned}$$

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Question 1

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$, so

$$\begin{aligned} \sec\left(\cos^{-1}\left(\frac{1}{2}\right)\right) &= \sec\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\cos\left(\frac{\pi}{6}\right)} \\ &= \frac{1}{1/2} \\ &= 2 \end{aligned}$$

Question 2

Since the limit as $x \to 0^+$ is zero in both the numerator and the denominator, we use L'Hopital's rule:

$$\begin{aligned} \lim_{x\to 0^+} \frac{e^x - 1}{\sin(x)} &= \lim_{x\to 0^+} \frac{e^x}{\cos(x)} \\ &= \frac{e^0}{\cos(0)} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$\begin{aligned} x &= 2y^3 + 1 \\ x - 1 &= 2y^3 \\ \frac{x-1}{2} &= y^3 \\ \sqrt[3]{\frac{x-1}{2}} &= y = f^{-1}(x) \end{aligned}$$

Question 4

We proceed by substitution, letting $u = 2x$ and $du = 2dx$:

$$\begin{aligned} \int e^{2x} dx &= \int e^u \frac{1}{2} du \\ &= \frac{1}{2} e^u + C \\ &= \frac{1}{2} e^{2x} + C \end{aligned}$$

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Question 1

$\cot^{-1}(1) = \frac{\pi}{2}$, so

$$\begin{aligned} \tan\left(\cot^{-1}(1)\right) &= \tan\left(\frac{\pi}{2}\right) \\ &= 1 \end{aligned}$$

Question 2

Since the limit as $x \to 0^+$ is zero in both the numerator and the denominator, we use L'Hopital's rule:

$$\begin{aligned} \lim_{x \to 0^+} \frac{e^x - 1}{x} &= \lim_{x \to 0^+} \frac{e^x}{1} \\ &= \frac{e^0}{1} \\ &= 1 \end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$\begin{aligned} x &= 2y^2 \\ \frac{x}{2} &= y^2 \\ \sqrt{\frac{x}{2}} &= |y| \\ \sqrt{\frac{x}{2}} &= y = f^{-1}(x) \end{aligned}$$

The choice of positive square root is made to give us the inverse on $(0, \infty)$.

Question 4

We know that:

$$\int \frac{dx}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

So by setting $u = x$ and $a = \sqrt{2}$, we obtain:

$$\begin{aligned} \int 2dx\sqrt{2-x^2} &= 2\int dx\sqrt{a^2-u^2} \\ &= 2\sin^{-1}\left(\frac{u}{a}\right) + C \\ &= 2\sin^{-1}\left(\frac{x}{\sqrt(2)}\right) + C \end{aligned}$$

1 PM

Question 1

$\tan^{-1}(1) = \frac{\pi}{2}$, so

$$\begin{aligned} \cot\left(\tan^{-1}(1)\right) &= \cot\left(\frac{\pi}{2}\right) \\ &= \frac{1}{\tan\left(\frac{\pi}{2}\right)} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

Question 2

Since the limit of the left term is 0 as $x \to 0^+$, while the limit of the right term is $-\infty$, we re-write as a fractional limit:

$$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{1/x}$$

Since the limits in the numerator and the denominator of this fraction are both zero, we can use L'Hopital's rule:

$$\begin{aligned} \lim_{x\to 0^+} \frac{\ln(x)}{1/x} &= \lim_{x\to 0^+} \frac{1/x}{-1/x^2} \\ &= \lim_{x\to 0^+} -\frac{x^2}{x} \\ &= \lim_{x\to 0^+} -x \\ &= 0 \end{aligned}$$

The evaluation at 0 follows from continuity.

Question 3

We proceed by switching x and y, and solving for the new y:

$$
\begin{aligned}
x &= y^3 + 2 \\
x - 2 &= y^3 \\
\sqrt[3]{x-2} &= y = f^{-1}(x)
\end{aligned}
$$

Question 4

Since the exponent in the first factor is e^x, we can let $u = e^x$ and $du = e^x dx$.

$$
\begin{aligned}
\int e^{e^x} e^x dx &= \int e^u e^x dx \\
&= \int e^u du \\
&= e^u + C \\
&= e^{e^x} + C
\end{aligned}
$$