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Question 1

Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{3^{n+1}(n + 1)!}{(2n + 3)!} \cdot \frac{(2n + 1)!}{3^n!} = \lim_{n \to \infty} \frac{3n}{(2n + 3)(2n + 2)} = 0 < 1
\]
so the series converges.

Question 2

We can rewrite this series as the geometric series \( \sum_{n=1}^{\infty} (2x)^n \) which converges when \(|2x| < 1\) or when \(|x| < \frac{1}{2}\). Thus the radius of convergence is \(\frac{1}{2}\).

Question 3

This is just the sum of a geometric series starting at \(n = 2\) with \(r = -1/3\), so the sum is
\[
\frac{(-\frac{1}{3})^2}{1 + \frac{1}{3}} = \frac{1}{12}.
\]

Question 4

Applying the ratio test we have
\[
\lim_{n \to \infty} \frac{x^{n+2}}{(n + 2)!} \cdot \frac{(n + 1)!}{x^{n+1}} = \lim_{n \to \infty} \frac{x}{n + 2} = 0 < 1.
\]
Since this holds for all values of \(x\), the radius of convergence is infinite, and thus the interval of convergence is \((-\infty, \infty)\).

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Question 1

Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{2^{n+1}(n + 1)!}{(2n + 2)!} \cdot \frac{(2n)!}{2^n!} = \lim_{n \to \infty} \frac{2(n + 1)}{(2n + 2)(2n + 1)} = 0 < 1,
\]
so the series converges.

Question 2

Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{2^{n+1}x^{n+1}}{(n + 1)!} \cdot \frac{n!}{2^n x^n} = \lim_{n \to \infty} \frac{2x}{n + 1} = 0 < 1.
\]
Since this holds for all values of \(x\), the radius of convergence is infinite.
Question 3
First note that by plugging 1 into the Taylor series for $e^x$, we have

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e.$$  

By subtracting the first term of the series and multiplying by 2 we get

$$\sum_{n=1}^{\infty} \frac{2}{n!} = 2(e - 1).$$

Question 4
We can rewrite this series as the geometric series $\sum_{n=1}^{\infty} (2x)^n$ which converges when $|2x| < 1$ or when $|x| < \frac{1}{2}$. When $x = \frac{1}{2}$, we have $\sum_{n=1}^{\infty} 1$ which clearly does not converge, and when $x = -\frac{1}{2}$ we have $\sum_{n=1}^{\infty} (-1)^n$ which also clearly does not converge. Therefore the interval of convergence is $(-\frac{1}{2}, \frac{1}{2})$.

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Question 1
Applying the ratio test, we have

$$\lim_{n \to \infty} \frac{(2n + 3)!}{(n + 1)^{n+1}} \cdot \frac{n^n}{(2n + 1)!} = \lim_{n \to \infty} \frac{(2n + 3)(2(n + 1))n^n}{(n + 1)(n + 1)^n} = \lim_{n \to \infty} 4n + 6 \cdot \frac{n}{n + 1} = \lim_{n \to \infty} \frac{4n + 6}{e} = \infty.$$  

Therefore the series diverges.

Question 2
Applying the ratio test, we have

$$\lim_{n \to \infty} \frac{(n + 1)!x^{n+1}}{2^{n+1} \cdot n!x^n} = \lim_{n \to \infty} \frac{nx}{2} = \infty$$  

for all values of $x$ except for $x = 0$. Therefore the radius of convergence is 0.

Question 3
Note that we can rewrite the series as a two times a geometric series. Evaluating, we get

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = 2 \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = 2 \cdot \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 4.$$
Question 4
Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{(n + 1)x^{n+1}}{nx^n} = \lim_{n \to \infty} \frac{n + 1}{n} \cdot \lim_{n \to \infty} x = x
\]
which is less than one exactly when \( x < 1 \). Therefore the radius of convergence is 1, and
all that remains is to check \( x = 1 \) and \( x = -1 \). At \( x = 1 \) we get the series \( \sum_{n=2}^{\infty} n \) which clearly diverges, and similarly at \( x = -1 \) we get the series \( \sum_{n=2}^{\infty} (-1)^n n \) which also clearly diverges since the terms do not go to 0. Therefore the interval of convergence is \((-1, 1)\).

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Question 1
Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{(2n + 2)!}{2^{n+1}(n + 1)!} \cdot \frac{2^n n!}{(2n)!} = \lim_{n \to \infty} \frac{(2n + 2)(2n + 1)}{2(n + 1)} = \infty,
\]
so the series diverges.

Question 2
Applying the ratio test, we have
\[
\lim_{n \to \infty} \frac{3^{n+1}x^{n+1}}{(2n + 2)!} \cdot \frac{(2n)!}{3^nx^n} = \lim_{n \to \infty} \frac{3x}{(2n + 2)(2n + 1)} = 0 < 1
\]
for all values of \( x \), so the radius of convergence is infinite.

Question 3
We can index the series to start at \( n = 0 \) instead of \( n = 1 \), and rewriting it we get
\[
\sum_{n=1}^{\infty} \frac{3}{(n - 1)!} = \sum_{n=0}^{\infty} \frac{3}{n!} = 3 \cdot \sum_{n=0}^{\infty} \frac{1}{n!} = 3e.
\]
The last equality holds because \( \sum_{n=0}^{\infty} \frac{1}{n!} \) is just the Taylor series for \( e^x \) evaluated at \( x = 1 \).

Question 4
We can rewrite this series as the geometric series \( \sum_{n=3}^{\infty} (3x)^n \) which converges when \( |3x| < 1 \)
or when \( |x| < \frac{1}{3} \). When \( x = \frac{1}{3} \), we have \( \sum_{n=1}^{\infty} 1 \) which clearly does not converge, and when
\[ x = -\frac{1}{3} \] we have \( \sum_{n=1}^{\infty} (-1)^n \) which also clearly does not converge. Therefore the interval of convergence is \((-\frac{1}{3}, \frac{1}{3})\).

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**Question 1**

Note that \( \ln(x) \) is a strictly increasing function, so

\[ \lim_{n \to \infty} \ln(1 + n) = \infty. \]

Since the terms of the series do not go to 0, the series diverges.

**Question 2**

Applying the ratio test, we have

\[ \lim_{n \to \infty} \frac{2^{n+1}x^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{2^n x^n} = \lim_{n \to \infty} \frac{2x}{n+2} = 0 < 1. \]

Since this holds for all values of \( x \), the radius of convergence is infinite.

**Question 3**

We can rewrite the series as

\[ \sum_{n=1}^{\infty} \frac{2}{3^n} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{3^n} = 2 \cdot \frac{\frac{1}{3}}{1 - \frac{1}{3}} = 1 \]

where the last equality holds because \( \sum_{n=1}^{\infty} \frac{1}{3^n} \) is a geometric series.

**Question 4**

Applying the ratio test, we have

\[ \lim_{n \to \infty} \frac{x^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{x^n} = \lim_{n \to \infty} \frac{x}{(2n+2)(2n+1)} = 0 < 1. \]

Since this holds for all values of \( x \), the radius of convergence is infinite. Therefore the interval of convergence is \((-\infty, \infty)\).
1 PM

Question 1
This series will diverge because the terms do not go to 0. For example,
\[ \lim_{n \to \infty} \frac{n^n}{n!} \geq n \]
for all \( n \), and \( \lim_{n \to \infty} n = \infty \) so \( \lim_{n \to \infty} \frac{n^n}{n!} = \infty \) as well.

Question 2
By factoring out a 3 from the numerator we can rewrite the series as a geometric series. This gives us
\[ \sum_{n=0}^{\infty} \frac{3^{n+1}x^n}{2^n} = 3 \cdot \sum_{n=0}^{\infty} \left( \frac{3x}{2} \right)^n \]
which converges only when \( \frac{3x}{2} < 1 \) which happens when \( x < \frac{2}{3} \). Therefore the radius of convergence is \( \frac{2}{3} \).

Question 3
If we factor out 5 from the numerator, then we get five times the Taylor series for \( e^x \) evaluated at 1. However since the sum starts at \( n = 2 \), we have to subtract the first two terms. Therefore we have
\[ \sum_{n=2}^{\infty} \frac{5}{n!} = 5 \cdot \sum_{n=2}^{\infty} \frac{1}{n!} = 5(e - 1 - 1) = 5(e - 2). \]

Question 4
Applying the ratio test, we have
\[ \lim_{n \to \infty} \frac{(n + 1)^2 x^{n+1}}{n^2 x^n} = \lim_{n \to \infty} \frac{(n + 1)^2}{n^2} \cdot \lim_{n \to \infty} x = x \]
which is less than one exactly when \( x < 1 \). Therefore the radius of convergence is 1, and all that remains is to check \( x = 1 \) and \( x = -1 \). At \( x = 1 \) we get the series \( \sum_{n=2}^{\infty} n^2 \) which clearly diverges, and similarly at \( x = -1 \) we get the series \( \sum_{n=2}^{\infty} (-1)^n n^2 \) which also clearly diverges since the terms do not go to 0. Therefore the interval of convergence is \((-1, 1)\).