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Question 1

Find the Taylor series for $f(x) = \ln(x)$ about the point $x = 2$. We start by looking at derivatives.

$$\begin{aligned} f(2) &= \ln(2) \\ f'(2) &= \frac{1}{2} \\ f''(2) &= \frac{-1}{4} \\ f'''(2) &= \frac{2}{8} \end{aligned}$$

We can see a pattern in every term except for the first, and so we can write the Taylor series as

$$\begin{aligned} \ln(x) &= \ln(2)x^0 + \sum_{n=1}^{\infty} \frac{(n-1)!(-1)^{n+1}}{n!2^n} x^n \\ &= \ln(2) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n} x^n \end{aligned}$$

Question 2

$$|2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned} \frac{1-i}{2-2i} &= \frac{1-i}{2-2i}\frac{2+2i}{2+2i} \\ &= \frac{2+2i-2i-2i^2}{4-4i^2} \\ &= \frac{4}{8} = \frac{1}{2} \end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned} (1-i)(2-2i) &= (1-2i-2i+2i^2) \\ &= (-1-4i) \end{aligned}$$

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Question 1

Find the Taylor series for $f(x) = \frac{x}{1+x}$ about the point $x = 0$. We start by looking at derivatives.

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 1 \\ f''(0) &= -2 \\ f'''(0) &= (-2)(-3) \\ f''''(0) &= (-2)(-3)(-4) \end{aligned}$$

We can see a pattern in every term except for the first, and so we can write the Taylor series as

$$\begin{aligned} \frac{x}{1+x} &= 0x^0 + \sum_{n=1}^{\infty} \frac{(n!(-1)^{n+1}}{n!} x^n \\ &= \sum_{n=1}^{\infty} (-1)^{n+1} x^n \end{aligned}$$

Question 2

$$|4+2i| = \sqrt{4^2+2^2} = \sqrt{20}$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned} \frac{2+2i}{2+2i} &= \frac{2+2i}{2+2i}\frac{2-2i}{2-2i} \\ &= \frac{4-4i^2}{4-4i^2} \\ &= \frac{8}{8} = 1 \end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned} (2+i)(2+2i) &= (4+4i+2i+2i^2) \\ &= (2+6i) \end{aligned}$$

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Question 1

Find the Taylor series for $f(x) = \sqrt{x+1}$ about the point $x = 0$. We start by looking at derivatives.

$$\begin{aligned} f(0) &= 1 \\ f'(0) &= \frac{1}{2} \\ f''(0) &= \frac{1}{2}\frac{-1}{2} \\ f'''(0) &= \frac{1}{2}\frac{-1}{2}\frac{-3}{2} \\ f''''(0) &= \frac{1}{2}\frac{-1}{2}\frac{-3}{2}\frac{-5}{2} \end{aligned}$$

We can see a pattern in every term except for the first, and so we can write the Taylor series as

$$\begin{aligned} \sqrt{x+1} &= 1x^0 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)!!}{n!(2n-1)2^n} x^n \\ &= 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-3)!!}{n!2^n} x^n \end{aligned}$$

Question 2

$$|2-3i| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned} \frac{2+2i}{1+i} &= \frac{2+2i}{1+i}\frac{1-i}{1-i} \\ &= \frac{2-2i^2}{1-i^2} \\ &= \frac{4}{2} = 2 \end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned} (2+2i)(1+i) &= (2+2i+2i+2i^2) \\ &= (0+4i) \end{aligned}$$

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Question 1

Find the Taylor series for $f(x) = xe^x$ about the point $x = 0$. We start by looking at derivatives.

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 1 \\ f''(0) &= 2 \\ f'''(0) &= 3 \\ f''''(0) &= 4 \end{aligned}$$

We can see a pattern in every term, and so we can write the Taylor series as

$$\begin{aligned} xe^x &= \sum_{n=0}^{\infty} \frac{n}{n!} x^n \\ &= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} x^n \end{aligned}$$

Question 2

$$\overline{2-3i} = 2-(-3)i = 2+3i$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned} \frac{3+2i}{2+2i} &= \frac{3+2i}{2+2i}\frac{2-2i}{2-2i} \\ &= \frac{6-6i+4i-4i^2}{4-4i^2} \\ &= \frac{10-2i}{8} = \frac{5}{4} - \frac{1}{4}i \end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned} (3+2i)(2+2i) &= (6+6i+4i+4i^2) \\ &= (2+10i) \end{aligned}$$

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Question 1

Find the Taylor series for $f(x) = x\sin(x^2)$ about the point $x = 0$. We start by looking at the Taylor series for $\sin(x)$ and substituting.

$$\begin{aligned}
\sin(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!} x^{2n+1} \\
\sin(x^2) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!} (x^2)^{2n+1} \\
x^2 \sin(x^2) &= x^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!} x^{4n+2} \\
x^2 \sin(x^2) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!} x^{4n+4}
\end{aligned}$$

Question 2

$$\overline{3+2i} = 3 - 2i$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned}
\frac{3+i}{3+2i} &= \frac{3+i}{3+2i}\frac{3-2i}{3-2i} \\
&= \frac{9-6i+3i-2i^2}{9-4i^2} \\
&= \frac{11-3i}{13} = \frac{11}{13} - \frac{3}{13}i
\end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned}
(3+2i)(3+i) &= (9+3i+6i+2i^2) \\
&= (7+9i)
\end{aligned}$$

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Question 1

Find the Taylor series for $f(x) = \frac{1}{\sqrt{x}}$ about the point $x = 1$. We start by looking at derivatives.

$$\begin{aligned} f(1) &= 1 \\ f'(1) &= \frac{-1}{2} \\ f''(1) &= \frac{-1}{2}\frac{-3}{2} \\ f'''(1) &= \frac{-1}{2}\frac{-3}{2}\frac{-5}{2} \end{aligned}$$

We can see a pattern in every term except the first, and so we can write the Taylor series as

$$\frac{1}{\sqrt{x}} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)!!}{n!2^n} x^n$$

Question 2

$$\overline{i+3} = -i + 3 = 3 - i$$

Question 3

In order to do complex division, we apply multiply numerator and denominator by the conjugate of the denominator:

$$\begin{aligned} \frac{1+2i}{2+2i} &= \frac{1+2i}{2+2i}\frac{2-2i}{2-2i} \\ &= \frac{2-2i+4i-4i^2}{4-4i^2} \\ &= \frac{6+2i}{8} = \frac{3}{4} + \frac{1}{4}i \end{aligned}$$

Question 4

We foil the two terms and then reduce any powers of i:

$$\begin{aligned} (1+2i)(2+2i) &= (2+2i+4i+4i^2) \\ &= (-2+6i) \end{aligned}$$