

de L'Hôpital's rule

Let $\lim_{x \rightarrow *}$ denote one of the following:

- (1) a classical limit - $\lim_{x \rightarrow a}$, where a is a number,
- (2) a one sided limit - $\lim_{x \rightarrow a^+}$ or $\lim_{x \rightarrow a^-}$,
- (3) or a limit at infity - $\lim_{x \rightarrow +\infty}$ or $\lim_{x \rightarrow -\infty}$,

respectively.

For each of these cases let I denote:

- (1) union of two open intervals adjacent to a (i.e., the interval (b, c) take away a , where $b < a < c$),
- (2) open interval with a as one of it endpoints (i.e., either $I = (a, b)$, or $I = (b, a)$),
- (3) or an open half line (i.e., $I = (b, +\infty)$, or $I = (-\infty, b)$),

respectively.

Assume f and g are differentiable on I , and that $g' \neq 0$ on I .

Assume $\lim_{x \rightarrow *} f(x) = \lim_{x \rightarrow *} g(x) = 0$, or, altrnatively, that $\lim_{x \rightarrow *} f(x) = \lim_{x \rightarrow *} g(x) = \pm\infty$.

Then,

$$\lim_{x \rightarrow *} \frac{f(x)}{g(x)} = \lim_{x \rightarrow *} \frac{f'(x)}{g'(x)},$$

provided the limit on the right hand side exists (this includes the case of the RHS limit being equal to $+\infty$ or $-\infty$).