MATH 416, HW 1, FALL 2014

1. Implement in Matlab the Gram-Schmidt orthogonalization algorithm.

2. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $x = (1;0;0;0)$, $y = (1;0;1;0)$, and $z = (1;1;1;0)$, using both, your software and "by hand".

3. Let $\langle u, v\rangle = \sum_{i=1}^{d} u_i v_i$ be the inner product on the d-dimensional Euclidean vector space $\mathbb{R}^d$. What is the relation of this inner product and the angle between vectors u and v in $\mathbb{R}^d$?

4. Describe the sets of vectors $x \in \mathbb{R}^2$, for which $\|x\|_p = r$, for any $r > 0$, where $p = 1, 2, \infty$. Use this description to find a vector $z \in \mathbb{R}^2$ such that $\|z\|_2 = 1$ and $\|z\|_1$ is as large as possible. What is this maximal value of $\|z\|_1$?

5. Plot in Matlab unit discs in $\mathbb{R}^2$ for the norms: $\|...\|_p$ for $p_1 = 1.2$ and $p_2 = 2.5$.