MATH 416, HW 2, FALL 2014

1. We say that an infinite collection of vectors $\{e_1, \dots, e_n, \dots\} \subset \mathbb{R}^d$, $n \geq d$ is a spanning set for $\mathbb{R}^d$ if every vector in $\mathbb{R}^d$ can be represented as a finite linear combination of vectors from the set $\{e_1, \dots, e_n, \dots\}$. We say that a collection of vectors $\{f_1, \dots, f_n, \dots\} \subset \mathbb{R}^d$, $n \geq d$ is a finite frame for $\mathbb{R}^d$ if there exist constants $A, B > 0 (A < B)$ such that for every vector $x \in \mathbb{R}^d$ the following holds:

$$A\|x\|_2^2 \leq \sum_{n=1}^{\infty} |\langle x, f_k \rangle|^2 \leq B\|x\|_2^2.$$

a) Are infinite spanning sets necessarily frames for $\mathbb{R}^d$? Prove or provide a counterexample.

b) Is every inifinite frame necessarily a spanning set for $\mathbb{R}^d$? Prove or provide a counterexample.

2. Show that the collection of vectors $(0,1), (\sqrt{3}/2, -1/2), (-\sqrt{3}/2, -1/2)$ in $\mathbb{R}^2$ is a tight frame (i.e., a frame with the lower frame bound A equal to the upper frame bound B). Find its frame constant.

3. Provide your own (and interesting) example of a tight frame for $\mathbb{R}^3$.

4. Show that $\{(1,0,0), (2,1,0), (3,2,1), (4,3,2)\}$, is a frame for $\mathbb{R}^3$.