MATH 401, HW 8, FALL 2015

1. (**10 points**) Adapt the proof of Theorem 5.7 from the textbook to show the following result:

Let $v_1, v_2, ..., v_k$, $k \leq n$, be a set of eigenvectors of a given $n \times n$ matrix A, such that for any eigenvalue λ of A, the subset of all these vectors belonging to λ is linearly independent. Then, the set of vectors $v_1, v_2, ..., v_k$ is linearly independent.

Please note that the above result is the last missing piece from our proof of Theorem 5.9, about a matrix A being diagonalizable if and only if the geometric multiplicity of every eigenvalue equals its algebraic multiplicity.

2. (**5 points**)Section 5.3, Problem 8.

3. (**10 points**) Recall that trace of a matrix is the sum of diagonal elements: $Tr(A) = \sum_{i=1}^{n} A_{i,i}$. Find the trace for all matrices in Section 5.1 Exercise 1. Next find all eigenvalues of these matrices. Is there a relationship between the eigenvalues and traces?