Answer key for Problem 2

October 6, 2008

$$L = \int_0^2 \sqrt{f'(t)^2 + g'(t)^2} dt \qquad \text{(2 points)}$$

$$f'(t) = (\ln 2)2^t \sin t + 2^t \cos t; \qquad g'(t) = (\ln 2)2^t \cos t - 2^t \sin t \qquad \text{(4 points)}$$

$$\begin{aligned} f'(t)^2 + g'(t)^2 = & \quad [(\ln 2)^2 2^{2t} \sin^2 t + 2^{2t+1}(\ln 2)\sin t \cos t + 2^{2t}\cos^2 t] \\ & +[(\ln 2)^2 2^{2t} \cos^2 t - 2^{2t+1}(\ln 2)\sin t \cos t + 2^{2t}\sin^2 t] \\ = & \quad (\ln 2)^2 2^{2t} \sin^2 t + 2^{2t}\cos^2 t + (\ln 2)^2 2^{2t}\cos^2 t + 2^{2t}\sin^2 t \end{aligned}$$

(4 points)

$$L = \int_0^2 \sqrt{(\ln 2)^2 2^{2t}(\cos^2 t + \sin^2 t) + 2^{2t}(\cos^2 t + \sin^2 t)} dt = \int_0^2 2^t \sqrt{(\ln 2)^2 + 1}\, dt$$

(4 points)

$$L = \sqrt{(\ln 2)^2 + 1} \int_0^2 2^t dt = \sqrt{(\ln 2)^2 + 1} \cdot \left. \frac{2^t}{\ln 2} \right|_0^2 \qquad \text{(4 points)}$$

$$L = \frac{3\sqrt{(\ln 2)^2 + 1}}{\ln 2} \qquad \text{(2 points)}$$