141 Exam 2 Fall 08 Dr. Czaja   answer key number 1

1) Solve the differential equation $\frac{dy}{dx} = \frac{1}{1-x}y + \cos(x)$.

This is a linear first-order differential equation. Two paths to the integrating factor are shown below.

| | points | |
|---|---|---|
| standard form: $\frac{dy}{dx} - \frac{1}{1-x}y = \cos(x)$ | | standard form: $\frac{dy}{dx} + \frac{1}{x-1}y = \cos(x)$ |
| thus $P(x) = -\frac{1}{1-x}$ and $Q = \cos(x)$ | 1 | thus $P(x) = \frac{1}{x-1}$ and $Q = \cos(x)$ |
| $u = 1 - x$, $du = -dx$ | | $u = x - 1$, $du = dx$ |
| $S(x) = \int -\frac{1}{1-x}\,dx = -[-\ln(1-x)]$ | 2 | $S(x) = \int \frac{1}{x-1}\,dx = \ln(x-1)$ |
| integrating factor = $e^{\ln(1-x)} = 1-x$ | 3 | integrating factor = $e^{\ln(x-1)} = x-1$ |

There are also two ways to approach the integration. On the left the multiplication has been distributed and two integrals evaluated. On the right, one integration by parts has been performed.

| | points | |
|---|---|---|
| $(1-x)\frac{dy}{dx} - y = (1-x)\cos(x)$ | | $(x-1)\frac{dy}{dx} + y = (x-1)\cos(x)$ |
| $\int \frac{d}{dx}[(1-x)y]\,dx = \int \cos(x)\,dx - \int x\cos(x)\,dx$ | | $\int \frac{d}{dx}[(x-1)y]\,dx = \int (x-1)\cos(x)\,dx$ |
| $y = \frac{1}{1-x}\left[\int \cos(x)\,dx - \int x\cos(x)\,dx\right]$ | 5 | $y = \frac{1}{x-1}\int (x-1)\cos(x)\,dx$ |
| for the 2nd integral, using integration by parts: $u = x$, $du = 1\,dx$; $dv = \cos(x)$, $v = \sin(x)$ | 1 | for the 1st integral, using integration by parts: $u = x - 1$, $du = 1\,dx$; $dv = \cos(x)$, $v = \sin(x)$ |
| $y = \frac{\sin(x) - \left[x\sin(x) - \int \sin(x)\,dx\right]}{1-x}$ | 2 | $y = \frac{(x-1)\sin(x) - \int \sin(x)\,dx}{x-1}$ |
| $y = \frac{\sin(x) - x\sin(x) - \cos(x) + C}{1-x}$ | 3 | $y = \frac{x\sin(x) - \sin(x) + \cos(x) + C}{x-1}$ |
| Note "+ $C$" also must be "over $(1 - x)$". | 3 | Note "+ $C$" also must be "over $(x - 1)$". |

1st note: The two answers are equal. These solutions check out when $y'$ is found and inserted into the DE.

2nd note: alternate simplified version: $y = \sin x - \frac{\cos(x) + C}{1-x} = \sin x + \frac{\cos(x) + C}{x-1}$