Math 141, Fall 2011, Midterm 1
Problem 1)

(12 points)
The length \( L \) of a curve given parametrically by \((x(t), y(t))\) for \(a \leq t \leq b\) is
\[
\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt.
\]
Here, \(x(t) = e^t \sin(t)\) and \(y(t) = e^t \cos(t)\) for \(0 \leq t \leq \pi\).
So we have \(x'(t) = e^t \cos(t) + e^t \sin(t) = e^t (\cos(t) + \sin(t))\) and
\(y'(t) = e^t \cos(t) - e^t \sin(t) = e^t (\cos(t) - \sin(t))\) for \(0 \leq t \leq \pi\).
So \(L = \int_0^\pi e^t \sqrt{(e^t (\cos(t) + \sin(t)))^2 + (e^t (\cos(t) - \sin(t)))^2} \, dt\)

(8 points)
\[
= \int_0^\pi e^t \sqrt{2(\cos^2(t) + \sin^2(t))} \, dt,
\]
and since \(\cos^2(t) + \sin^2(t) = 1\), we have
\[
L = \int_0^\pi e^t \sqrt{2} \, dt = \sqrt{2}(e^\pi - e^0) = \sqrt{2}(e^\pi - 1).
\]

(5 points)
To find the starting and ending points of this curve, we simply plug in the first and last time values for \(t\) in \((x(t), y(t))\) to get that the starting point is \((x(0), y(0)) = (0, 1)\) and that the ending point is \((x(\pi), y(\pi)) = (0, e^{-\pi})\).