

$$1) \textcircled{A} \int_0^{\frac{\pi}{2}} \sin^2 t + \cos^3 t \, dt = \int_0^{\frac{\pi}{2}} \sin^2 t (1 - \sin^2 t) \cos t \, dt$$

$$u = \sin t \\ du = \cos t \, dt \quad \left. \vphantom{\int} \right] \text{5 points for correct } u \text{ substitution}$$

$$\begin{aligned} &= \int_0^1 u^2 (1 - u^2) \, du \\ &= \left( \frac{1}{3} u^3 - \frac{1}{5} u^5 \right) \Big|_0^1 \end{aligned} \quad \left. \vphantom{\int} \right] \text{2 points for correct integration}$$

$$= \frac{1}{3} - \frac{1}{5}$$

$$= \frac{2}{15} \quad \left. \vphantom{\int} \right] \text{1 point for correct answer + evaluating using limits of integration}$$

$$\textcircled{B} \int \frac{x}{\sqrt{4-x^2}} \, dx = -\frac{1}{2} \int \frac{1}{\sqrt{u}} \, du = -\sqrt{u} + C$$

5 points for correct integration

5 points for correct u substitution

$$\begin{cases} u = 4 - x^2 \\ du = -2x \, dx \\ -\frac{1}{2} du = x \, dx \end{cases}$$

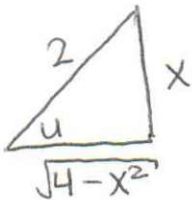
$$= -\sqrt{4-x^2} + C$$

1 point

5 points

Alternative soln

$$\int \frac{x}{\sqrt{4-x^2}} \, dx = \int \frac{2 \sin u}{\sqrt{4-4\sin^2 u}} (2 \cos u) \, du$$



$$\begin{aligned} \cos u &= \frac{x}{2} \\ 2 \sin u &= x & x^2 &= 4 \sin^2 u \\ 2 \cos u \, du &= dx \end{aligned}$$

7 points (triangle + substitution)

$$= 2 \int \frac{\sin u}{\cos u} \cos u \, du$$

$$= +2 \int \sin u \, du$$

$$= -2 \cos u + C$$

$$= -2 \frac{\sqrt{4-x^2}}{2} + C$$

$$= -\sqrt{4-x^2} + C$$

5 points for correct integration

3 pts for correct soln (using triangle to get soln in terms of x)  
1 point (of the 3)