Math 141 Midterm 4
Solution to number 1
1) Find the sum of the series \( \sum_{k=1}^{\infty} \frac{(-1)^k 4^{k-1}}{3^{2k}} \).

Asking for the sum of the series (as opposed to just whether or not it converged) should have been a hint that this series was either geometric or telescoping (these are most of the instances where we can find the sum exactly). It is clearly geometric with ratio \(-\frac{4}{9}\). The first term is \(-\frac{1}{9}\) and so the sum converges to \( \frac{-\frac{2}{9}}{1 - (-\frac{4}{9})} = \frac{1}{13} \).

As far as grading went, if the student realized it was geometric that was worth about 10 points, with the first term and ratio being worth 5 points each.

If the student didn’t realize it was geometric, the usual route was performing the ratio test. If done correctly, this was worth 5 points (even though that only tells you it converges and not the sum itself).