

# MATH141 - Mid Term Exam 1 - Problem 2

Let,

$$x(t) = t - \sin(t)$$

$$y(t) = -1 + \cos(t), \quad t \in [0, 2\pi]$$

Then,

$$x'(t) = 1 - \cos(t) \quad \boxed{2}$$

$$y'(t) = -\sin(t) \quad \boxed{2}$$

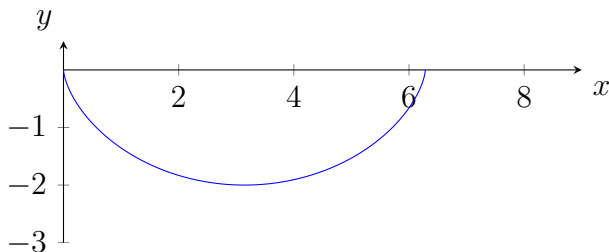
$$\begin{aligned} \text{Length of the curve} &= \int_0^{2\pi} \sqrt{x'(t)^2 + y'(t)^2} dt \quad \boxed{4} \\ &= \int_0^{2\pi} \sqrt{(1 - \cos(t))^2 + (-\sin(t))^2} dt \\ &= \int_0^{2\pi} \sqrt{2 - 2\cos(t)} dt \quad \boxed{2} \\ &= \int_0^{2\pi} \sqrt{4\sin^2\left(\frac{t}{2}\right)} dt \quad \boxed{2} \\ &= \int_0^{2\pi} \left| 2\sin\left(\frac{t}{2}\right) \right| dt \\ &= 4 \int_0^{\pi} \sin(t) dt \quad \boxed{2} \quad (\text{Any correct method of integration}) \\ &= 8 \quad \boxed{1} \end{aligned}$$

$$\text{Starting Point} = (x(0), y(0)) = (0, 0) \quad (\text{Corresponds to } t = 0) \quad \boxed{2}$$

$$\text{Ending Point} = (x(2\pi), y(2\pi)) = (2\pi, 0) \quad (\text{Corresponds to } t = 2\pi) \quad \boxed{2}$$

$$\text{Intermediate Point} = (x(\pi), y(\pi)) = (\pi, -2) \quad (\text{Corresponds to } t = \pi) \quad \boxed{2}$$

(Any correct intermediate point)



$\boxed{4}$