<u>Problem 2</u> : $\sum_{n\in\mathbb{N}} \frac{2^n}{n3^n} x^{2n}$

$a_n = \frac{2^n}{n3^n} |x|^{2n}$, $a_{n+1} = \frac{2^{n+1}}{3^{n+1}(n+1)} |x|^{2n+2}$ [1]

Applying the ratio test : [2]

$\frac{a_{n+1}}{a_n} = \frac{2}{3} |x|^2 \frac{n}{n+1}$ [2]

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{3} |x|^2$ [3]

Or, applying the root test : [2]

$\sqrt[n]{a_n} = \frac{2}{3} \frac{|x|^2}{\sqrt[n]{n}}$ [2]

$\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{2}{3} |x|^2$ [3]

This limit must be $<1 \Rightarrow |x| < \sqrt{3/2}$

$\Rightarrow$ the radius of convergence is $\sqrt{3/2}$ [2]

$\Rightarrow$ the series converges absolutely on $-\sqrt{3/2} < x < \sqrt{3/2}$ [2]

At $x = \sqrt{3/2}$, the series is : $\sum_{n\in\mathbb{N}} \frac{2^n}{n3^n} (\sqrt{\frac{3}{2}})^{2n} = \sum_{n\in\mathbb{N}} \frac{1}{n} = \infty$ [2+2+2]

At $x = -\sqrt{3/2}$, the series is $\sum_{n\in\mathbb{N}} \frac{2^n}{n3^n} (-\sqrt{\frac{3}{2}})^{2n} = \sum_{n\in\mathbb{N}} \frac{1}{n} = \infty$. [2+2+3]