The tank has parabolic cross sections by design, since we obtain the tank by rotating the parabola $y = 4x^2$ around the $x$-axis for $x \in [0,1]$. Since $4x^2$ is strictly increasing on this interval, the tank height occurs at the right-hand endpoint, so $h = 4(1)^2 = 4$. The water is being pumped from a full tank to the top of the tank, and we are stopping when there is one foot of water left. The standard equation for work is so far

$$W = 62.5 \int_1^4 (4-y)A(y)dy$$

We need to compute the area of a given $y$-cross section. Since $y = 4x^2$, we have $x = \sqrt{y}/2$. The cross sections are clearly circular, and the radius is $x$, so $A(y) = \pi x^2 = \frac{\pi y}{4}$. This gives the integral

$$W = 62.5 \int_1^4 (4-y) \left(\frac{\pi y}{4}\right) dy = 62.5 \cdot \frac{9\pi}{4}$$

The integral itself is not hard to evaluate, so most of the points were in setting up the problem. The points were distributed as follows:

1. 5 points for having the correct formula for work.
2. 5 points for finding the tank height.
3. 5 points for having the correct limits of integration.
4. 5 points for computing the cross-sectional area correctly.
5. 5 points for evaluating the integral.

For each of these, I wrote $+k$ near the relevant part of your work if you got $k$ points for that part. But note that I interpret “having the correct formula for work” to mean that you understand the formula. Thus if you know the formula has an $(\ell - x)$ factor, I expect you to know what $\ell$ is. Also, if you did a volume integral to find the volume of the tank, that suggested you didn’t understand the formula, since the formula uses cross-sectional area and not total volume of the tank.

Common mistakes were to switch $x$ and $y$ for only some parts of the problem. For example, many students wrote down the correct formula for work with $x$ and $y$ switched, but had the cross-sectional area as $\pi x^2$, which should have really been $\pi y^2$ if $x$ and $y$ were switched.

Another common mistake was to compute work for a hemispherical tank rather than a tank with parabolic cross-sections. If you wrote $A(y) = \sqrt{r^2 - y^2}$ for some value of $r$, this is what you did.

1Note that we are integrating with respect to $y$ since the setup of the problem forced $y$ to be the vertical axis. If you like to integrate with respect to $x$, we can change the $x$’s and $y$’s as long as we remember to rotate the function $x = 4y^2$ around the $x$-axis.