

Grading scheme for Problem 3

$$f'(x) = \frac{d}{dx} \left( \frac{x}{1+x^2} \right) = \frac{1+x^2 - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \quad [6]$$

$$\Rightarrow f'(x) = 0 \text{ at } x = \pm 1 \quad [3]$$

$$\Rightarrow f'(x) \neq 0 \text{ for } x \in (-1, 1) \Rightarrow f \text{ is invertible / monotonic on } [-1, 1] \quad [3]$$

$$\frac{d}{dx} f^{-1}(x) \Big|_{x=0} = \frac{1}{\frac{d}{dx} f(x) \Big|_{x=f^{-1}(0)}} \quad [4]$$

$$\text{To get } f^{-1}(0) : \frac{x}{1+x^2} = 0 \Rightarrow x = 0 \Rightarrow f^{-1}(0) = 0 \quad [3]$$

$$\frac{d}{dx} f(x) \Big|_{x=0} = 1 \quad [3]$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left( \frac{x}{1+x^2} \right) = \frac{1}{1} = 1 \quad [3]$$