Midterm 2

3) Determine whether the function $f(x) = x + \sin(x)$ has an inverse. If so, give the domain and range.

(5 points) $f(x) = x + \sin(x)$, so $f'(x) = 1 + \cos(x)$.

(15 points) $f'(x) \geq 0$ and $f'(x) = 0$ for $x = \pi + 2k\pi$ (for integer $k$) which is a discrete set and hence $f$ is increasing. Thus, $f$ has an inverse.

(10 points) The domain and range of $f^{-1}$ are the range and domain (resp) of $f$. The domain of $f$ is all reals, hence so is the range of $f^{-1}$. Also, note that since $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = \infty$, and $f$ is continuous, the range of $f$ is all reals, hence so is the domain of $f^{-1}$. 