

MATH141 - Mid Term Exam 2 - Problem 4

Method I (By appropriately rearranging the integrand and then integration by parts)

$$\begin{aligned}\int_0^1 x^5 e^{x^3} dx &= \int_0^1 \left(\frac{1}{3}x^3\right) \left(3x^2 e^{x^3}\right) dx && \boxed{6} \\ &= \frac{1}{3}x^3 e^{x^3} \Big|_0^1 - \int_0^1 x^2 e^{x^3} dx && \boxed{5} + \boxed{5} \\ &= \left(\frac{e}{3} - 0\right) - \frac{1}{3} \int_0^1 3x^2 e^{x^3} dx && \boxed{2} + \boxed{1} \\ &= \frac{e}{3} - \frac{1}{3}e^{x^3} \Big|_0^1 && \boxed{3} \\ &= \frac{e}{3} - \frac{1}{3}(e^1 - e^0) && \boxed{2} \\ &= \frac{1}{3} && \boxed{1}\end{aligned}$$

Method II (By substitution and then integration by parts)

We substitute $u = x^3$ then $du = 3x^2 dx$. $\boxed{2} + \boxed{2}$

After appropriately changing the limits we have,

$$\begin{aligned}\int_0^1 x^5 e^{x^3} dx &= \frac{1}{3} \int_0^1 u e^u du && \boxed{4} \\ &= \frac{1}{3} \left[u e^u \Big|_0^1 - \int_0^1 e^u dx \right] && \boxed{5} + \boxed{5} \\ &= \left(\frac{e}{3} - 0\right) - \frac{1}{3}e^u \Big|_0^1 && \boxed{2} + \boxed{2} \\ &= \frac{e}{3} - \frac{1}{3}(e^1 - e^0) && \boxed{2} \\ &= \frac{1}{3} && \boxed{1}\end{aligned}$$

OR any other **mathematically correct** method.