Find the interval of convergence for the series
\[ \sum_{n=2}^{\infty} \frac{2x^n}{3^{n+1}n^2} \]

SOLUTION
1. Determine the radius of convergence [15pts distributed as follows: indicating an appropriate test - 2 pts, setting up the limit - 2pts, finding the limit - 8 pts, making the conclusion about the radius of convergence - 3 pts]

Option 1. Ratio test:
\[
\begin{align*}
r &= \lim_{n \to \infty} \left| \frac{2x^{n+1}}{3^{n+2}(n+1)^2} / \frac{2x^n}{3^{n+1}n^2} \right| = \lim_{n \to \infty} \frac{2|x|^{n+1}3^n+1n^2}{3^{n+2}(n+1)^22|x|^n} \\
&= \lim_{n \to \infty} \frac{|x|^{n+1}}{3(n+1)^2} = \frac{|x|}{3} \lim_{n \to \infty} \frac{n^2}{(n+1)^2} = \frac{|x|}{3} \lim_{n \to \infty} \frac{1}{(1 + \frac{1}{n})^2} = \frac{|x|}{3}
\end{align*}
\]
Series converges absolutely when \( r = \frac{|x|}{3} < 1 \), i.e. \(|x| < 3\) and diverges if \( r = \frac{|x|}{3} > 1 \), i.e. \(|x| > 3\), hence, \( R = 3 \).  

Option 2. Root test:
\[
\begin{align*}
r &= \lim_{n \to \infty} \sqrt[n]{\left| \frac{2x^n}{3^{n+1}n^2} \right|} = \lim_{n \to \infty} \frac{2^{1/n}|x|}{3^{1/n}3n^{2/n}} = \frac{|x|}{3}
\end{align*}
\]
Series converges absolutely when \( r = \frac{|x|}{3} < 1 \), i.e. \(|x| < 3\) and diverges if \( r = \frac{|x|}{3} > 1 \), i.e. \(|x| > 3\), hence, \( R = 3 \).

2. Check whether the series converges at the endpoints of the interval \((-R, R)\) (if \( R \) is not 0 or \( \infty \))(5pts)

First check \( x = 3 \): the series becomes
\[
\sum_{n=2}^{\infty} \frac{23^n}{3^{n+1}n^2} = \frac{2}{3} \sum_{n=2}^{\infty} \frac{1}{n^2},
\]
which is a multiple of the convergent series \((p\text{-series with } p = 2 > 1)\), hence, the series converges and \( x = 3 \) is included into the interval of convergence.
Now check $x = -3$: the series becomes

$$\sum_{n=2}^{\infty} \frac{2(-3)^n}{3^{n+1}n^2} = \frac{2}{3} \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2},$$

which is absolutely convergent for the reason described above. Hence, $x = -3$ is included into the interval as well.

The final answer is $I = [-3, 3]$. 