Exam 3 Problem 4:

First, let $f(x) = \frac{\log_3(\frac{1}{x})}{x}$, so $f(n) = a_n$ for all natural numbers $n \geq 1$. So by continuity of f, $\lim_{n\to\infty} \frac{\log_3(\frac{1}{n})}{n} = \lim_{x\to\infty} f(x)$.

Then, using L'Hopital's rule,

$$\lim_{x\to\infty} \frac{\log_3(\frac{1}{x})}{x} = \lim_{x\to\infty} -\frac{\frac{1/x^2}{(\ln 3)(1/x)}}{1} = \lim_{x\to\infty} -\frac{1}{x\ln 3} = 0$$

.

Alternatively, if f is rewritten as

$$\frac{\log_3(\frac{1}{x})}{x} = \frac{\log_3(1) - \log_3(x)}{x} = -\frac{\log_3(x)}{x},$$

then the calculation of derivatives for L'Hopital's rule is simpler in that $\frac{d}{dx}(\log_3(x)) = \frac{1}{x\ln 3}$.

- 5 points for comparing the limit of the sequence with the limit of a real-variable function
- 2 points for noting that L'Hopital's rule is to be used
- 12 points for correct application of L'Hopital's rule
  - (-1) point for sign error
  - (-2) points for leaving off $\ln 3$
  - (-6) points for division errors, i.e. exchanging numerator and denominator
- 6 points for simplifying and obtaining the correct solution
  - 3 points for getting that the limit is 0
  - 3 points for correct simplification