Math 141 Midterm 4 Question 4 Solution

December 8, 2014

Question: a) Write
$$\frac{2\sqrt{3}-2i}{-1+i\sqrt{3}}$$
in the polar form $re^{i\theta}$.
b)Find all fourth roots of 7, i.e., all complex number z such that $z^4 = 7$.
Solution:
a) Multiply the numerator and denominator by the complex conjugate of the denominator $-1 - i\sqrt{3}$ and simplify to
$$-\sqrt{3} - i \qquad \text{7 pts}$$
Now change into polar form. $r = \sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$ and so
$$-\sqrt{3} - i = 2(\cos(\theta) + i\sin(\theta))$$
Equate the real and imaginary pieces so $\cos(\theta) = \frac{-\sqrt{3}}{2}$ and $\sin(\theta) = -\frac{1}{2}$. Thus
$$r = 2, \theta = \frac{7\pi}{6} + 2\pi k \qquad \text{5 pts}$$
Therefore
$$\frac{2\sqrt{3}-2i}{-1+i\sqrt{3}} = 2e^{i(\frac{7\pi}{6}+2\pi k)} \qquad \text{3 pts}$$
b) Since 7 is on the real axis we can see that $r = 7$ and $\theta = 2\pi k$. Therefore
$$z^4 = 7e^{i2\pi k} \qquad \text{5 pts}$$
Thus
$$z = 7^{\frac{1}{4}} e^{i\frac{\pi}{2}k} \qquad \text{1 pt}$$
To find the four roots we let $k = 0, 1, 2, 3$. Our four roots are
$$z_1 = 7^{\frac{1}{4}} \qquad \text{1 pt}$$
$$z_2 = 7^{\frac{1}{4}} e^{i\frac{\pi}{2}} \qquad \text{1 pt}$$
$$z_3 = 7^{\frac{1}{4}} e^{i\pi} \qquad \text{1 pt}$$
$$z_4 = 7^{\frac{1}{4}} e^{i\frac{3\pi}{2}} \qquad \text{1 pt}$$