MATH 648M: Advanced Analytic Methods in Applied Mathematics
Department of Mathematics, UMCP **Spring 2009**
Homework 4 **Posted**: Wednesday, 03/25/09
Due: Tuesday, 04/07/09

Students who take the course for 3 credits should do all problems (10-12). Students who take the course for n credits ($n = 1$, 2) should do any n problems from problems 10-12.

10. (20pts) [Renormalized perturbation] This problem focuses on the motion of *two* classical particles of *unit mass* and positions $(x(t), y(t))$ in a two-body potential $V(x, y)$.

    (a)[5 pts] Write down the general equations of motion for this system.

    (b)[15 pts] Consider the case of two coupled harmonic oscillators, where $V(x, y) = 2^{-1}(\omega_0^2 x^2 + W_0^2 y^2 + \epsilon x^2 y^2)$; $\omega_0 > 0$ and $W_0 > 0$ are the (unperturbed) frequencies of the uncoupled oscillators. The initial conditions are $x(0) = 1$, $\dot{x}(0) = 0$, and $y(0) = 0$, $\dot{y}(0) = 1$. Apply the renormalized perturbation and determine approximately the actual frequencies of the motion and the positions $x_0(t)$ and $y_0(t)$. How is the frequency of each oscillator affected by their coupling ?

11. (20 pts) By applying the two-scale method, solve for u in the following inital-value problem:

$$\ddot{u} + u = \epsilon(\dot{u} - \frac{1}{3}\dot{u}^3);\ u(0) = 0,\ \dot{u}(0) = a;\ 0 < \epsilon \ll 1.$$

    In addition, answer the following question: What is the limit as $t \to \infty$ of your approximate solution ? Discuss the validity of your expansion.

12.](20 pts) [Homogenization in one space dimension.] Consider the problem

$$\partial_x(D\partial_x u^\epsilon) = \partial_t u^\epsilon + f(x, x/\epsilon), \quad 0 < x < 1,$$

    where $u^\epsilon = 0$ at $x = 0$, 1 and $u^\epsilon = 0$ at $t = 0$; assume $u^\epsilon = u^\epsilon(x, t)$ and $D = D(x, x/\epsilon)$.

    (a) (10 pts) Find the homogenized problem for the steady state ($\partial_t u^\epsilon \equiv 0$). **Hint:** Make suitable assumptions for u^ϵ and f.

    (b) (10 pts) Find the homogenized problem for the time-dependent solution. **Hint**: Define slow anf fast space and *time* variables.