1. Consider the function
   \[ f(z) = \frac{z^2 + z - 1}{z(z - 1)^2}. \]

   (a)[2pts] Find and classify all singular points of \( f(z) \).
   (b)[8pts] Compute the integral
   \[ I = \int_{\Gamma} f(z) \, dz, \]
   where the closed contour \( \Gamma \) is traversed once positively, in the following cases:
   b.i) \( \Gamma \) is the circle with center at \( z = 2 \) and radius \( 3/2 \);
   b.ii) \( \Gamma \) is circle with center at \( z = 2 \) and radius 100.

2. Consider the integral
   \[ H(z) = \frac{1}{2\pi i} \int_{C} \frac{e^\zeta + \zeta^{-1}}{\zeta - z} \, d\zeta, \]
   where \( C \) is the circle with center at \( z = 0 \) and radius 1, traversed once counterclockwise.
   Compute the following values:
   (a)[4pts] \( H(0) \);
   (b)[6pts] \( \lim_{z \to i} H(z) \) if \( z \) lies outside \( C \).

3. Consider the function
   \[ f(z) = \frac{z - 1}{3 - z}. \]

   (a)[8pts] Find the Taylor series for \( f(z) \) at \( z_0 = 0 \). What is the radius of convergence of this Taylor series? Explain carefully.
   (b)[2pts] Consider the function \( g(z) = e^{f(z)} \). What kind of isolated singularity of \( g(z) \) is the point \( z_0 = 3 \)? Explain carefully.

4. [10pts] Consider the function
   \[ f(z) = \frac{1}{z(z - i)}. \]
   Find the Laurent series for \( f(z) \) at \( z_0 = 0 \) in the annulus \( 1 < |z| < \infty \).