Answer all questions. Make sure that you explain all your steps and justify your answers.

1. (15pts) By the theory of residues, compute the integral

\[ I = \int_0^\pi \frac{d\theta}{3 + 2\cos \theta}. \]

2. (15pts) By using contour integration, compute the integral

\[ I = \text{p.v.} \int_{-\infty}^{\infty} \frac{xe^{3x}}{x^2 - 4} \, dx = \lim_{\rho \to \infty} \left( \int_{-\rho}^{-2-r_1} + \int_{2-r_2}^{2+r_2} + \int_{2+r_2}^{\rho} \right) \frac{xe^{3x}}{x^2 - 4} \, dx. \]

3. (10pts) Consider the function

\[ f(z) = \frac{\cos(\lambda/z^2)}{1 - \mu z}. \]

(a)[2pts] Set \( \lambda = 0 \) and \( \mu = 1 \). Characterize the singular point \( z = 1 \). Expand \( f(z) \) in powers of \( z \) in the disk \(|z| < 1\).

(b)[3pts] Set \( \lambda = 1 \) and \( \mu = 0 \). Characterize the singular point \( z = 0 \). Expand \( f(z) \) in Laurent series around \( z_0 = 0 \) in the annulus \(|z| > 0\). What is the residue of \( f(z) \) at \( z = 0 \)?

(c)[5pts] Now set \( \lambda = 1 \) and \( \mu = 1 \). By the theory of residues, compute the integral

\[ I = \frac{1}{2\pi i} \oint_C f(z) \, dz, \]

if the closed contour \( C \) is the circle \(|z| = 1/3\) traversed once counterclockwise.

\textbf{Hint on 3(c):} There are at least two ways to do this problem. One way is to pursue calculation of one residue only (which one?). If you follow this way, then you have to multiply two infinite series expansions; and be very careful about how to compute the desired coefficient \( a_{-1} \) (residue) term.