1. [5 pts] Define the function $H$ by $H(n) = \lfloor n/2 \rfloor = \text{the largest } k \leq n/2$. Prove that $H$ is recursive.

2. [5 pts] Assume that $X \subseteq \mathbb{N}$ is recursive. Define the function $F$ by $F(n) = \text{the number of elements in } X \text{ which are } < n$. Prove that $F$ is recursive.
   Hint: Use primitive recursion.

3. [5 pts] Let $X = \{0, 1, (1 + 2), (1 + 2 + 3), \ldots, (1 + 2 + \cdots + n), \ldots \}$. Prove that $X$ is recursive.

4. [5 pts] Show directly that the function $F$ defined by $F(n) = \lceil \frac{n \cdot n}{n} \rceil$ is recursive.
   Warning: $\frac{n \cdot n}{n} \neq \bar{n} \cdot \bar{n}$.

**NOTE:** Your solutions must include enough detail to justify your conclusions.