1. Let $\leq$ be the usual order on $\mathbb{N}$.

   [10 pts] (a) Explicitly define a linear order $\leq^*$ on $\mathbb{N}$ which is not a well order. Justify your answer.

   [15 pts] (b) Explicitly define a well order $\leq^\#$ on $\mathbb{N}$ such that $(\mathbb{N}, \leq) <_\omega (\mathbb{N}, \leq^\#)$. Justify your answer.

2. [20 pts] Let $U$ be a well ordered set. Prove, by induction, that the following holds for every $x \in U$:

   there is some $z \in U$, where $z$ is either $0_U$ or a limit point of $U$, and some $n \in \mathbb{N}$ such that $x = S^n(z)$.

   $S^n$ is defined by recursion on $\mathbb{N}$: $S^0(z) = z$, $S^{n+1}(z) = S(S^n(z))$.

3. [20 pts] Assume that for every non-empty set $A$ and for every equivalence relation $E$ on $A$ there is some function $f$ whose domain is the set of equivalence classes of $E$ and is such that for every $a \in A$, $f([a/E]) \in [a/E]$. Prove that the Axiom of Choice holds.

   Hint: It suffices to show the existence of choice sets.

4. [15 pts] Prove that $\mathbb{N} \leq_c A$ for every infinite set $A$ using Dependent Choice, but not the full Axiom of Choice.

5. [20 pts] (AC) Assume that there is a surjection $g$ of $A$ onto $B$. Prove that $B \leq_c A$.

**NOTE:** Your solutions must include enough detail to justify your conclusions.