1. [5 pts] Define $[\mathbb{N}]^2 = \{X \subseteq \mathbb{N} \mid \#(X) = 2\}$. Define a well order of $[\mathbb{N}]^2$, and prove it is a well order.

2. Let $A$ be a well orderable set.

   [5 pts] (a) Assume that $A$ is infinite, and let $a^*$ be some object not in $A$. Prove that $(A \cup \{a^*\}) =_c A$.

   [5 pts] (b) Assume that $B$ is also well orderable. Prove that $(A \cup B)$ is well orderable.

3. [5 pts] Prove that for every set $A$ there is a well ordered set $V$ such that there is no surjection $\pi$ of $A$ onto $V$.

4. [5 pts] Prove that if $A \leq_c B$ then $\chi(A) \leq_o \chi(B)$.

5. [5 pts] Let $U$ be a well ordered set. Prove that if $U \leq_c A$ then $U <_o \chi(A)$.

**NOTE:** Your solutions must include enough detail to justify your conclusions.