1. [15] Find all incongruent solutions to $20x \equiv 12 \pmod{36}$. Your answers should be least non-negative residues.

2. [15] Find the smallest positive integer which is divisible by 7, leaves a remainder of 3 when divided by 5, and leaves a remainder of 2 when divided by 8. [No credit for guessing or random search!]


4. [10] (a) Assume that $(a, 12) = 1$. Show that $a^6 \equiv a^2 \pmod{12}$.
       (b) Assume that $(a, 60) = 1$. Show that $a^6 \equiv a^2 \pmod{60}$.

5. [15] Show that $\phi(3n) = 2\phi(n)$ if $3 \nmid n$ but that $\phi(3n) = 3\phi(n)$ if $3|n$.

6. [10] (a) Prove that if $(2^m - 1)$ is prime then $2^{m-1}(2^m - 1)$ is perfect.
       (b) Let $m$ be a perfect number and let $k > 1$. Show that $\sigma(km) > 2km$. 