MATH 406 – HOMEWORK 9
(due Monday 7 May 2012)

1. Use the arithmetic properties of the Legendre symbol to evaluate \( \left( \frac{77}{103} \right) \).

2. Let \( p \) and \( q \) be odd primes and assume that \( p \equiv q \pmod{20} \). Prove that \( \left( \frac{5}{p} \right) = \left( \frac{5}{q} \right) \).

3. Show that if \( x, y, z \) is a primitive Pythagorean triple then either \( x \) or \( y \) is divisible by 3.

4. Find formulas for the integers of all (primitive) Pythagorean triples \( x, y, z \) with \( z = y + 1 \).

5. Show that the Diophantine equation \( x^4 + 3y^4 = z^2 \) has infinitely many solutions.

**NOTE:** Explain your work clearly. Your solutions must include enough detail to justify your conclusions.