AMSC/CMSC 460: HW #2
Due: Tuesday 2/9/17 (in class)

$\boxed{\text{Please submit the solution to at least one problem in LaTeX.}}$

1. Use the bisection method to find solutions, accurate to within 10^{-5} for the following problems:

   (a) $x - 2^{-x} = 0, \quad$ for $0 \le x \le 1$

   (b) $e^x - x^2 + 3x - 2 = 0, \quad$ for $0 \le x \le 1$.

2. Perform four iterations of Newton's method for the polynomial $p(x) = 4x^3 - 2x^2 + 3$ starting with $x_0 = -1$.

3. Use both Newton's method and the Secant method to find solutions accurate to within 10^{-5} for the following problems:

   (a) $e^x + 2^{-x} + 2\cos x - 6 = 0, \quad$ for $1 \le x \le 2.$,

   (b) $\ln(x-1) + \cos(x-1) = 0, \quad$ for $1.3 \le x \le 2$.

   Compare the number of iterations you had to use with each method to obtain a root with desired accuracy.

4. The function
$$f(x) = \frac{x}{\sqrt{1+x^2}}$$
   has a unique root $x = 0$.

   (a) Show that Newton's method gives $x_{k+1} = -x_k^3$. Conclude that the method succeeds in approximating the root if and only if $|x_0| < 1$.

   (b) Draw graphs to illustrate the first few iterates when $x_0 = .5$ and $x_0 = 1.5$.

5. What is the purpose of the following iterative process?
$$x_{n+1} = 2x_n - x_n^2 y, \quad n \ge 0.$$
   <u>Hint:</u> Fix y and assume that the method converges, i.e., $x_n \to a$ as $n \to \infty$. What is a?
   To check your answer, compute ten iterations of the method when $x_0 = 0.5$ and $y = 0.1$.
   <u>Difficult Bonus:</u> Identify it as the Newton iteration for a certain function.

6. Read Sections 4.5, 4.6, and 4.7 from Chapter 4 (Zeros and Roots) in Cleve Moler's book.

   (a) Explain in your words how the algorithm "zeroin" works

   (b) Solve problem 4.15(a).