AMSC/CMSC 460: HW #3
Due: Thursday 2/16/17 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Let $f(x) = x^3$. Find the second Taylor polynomial $P_2(x)$ about $x_0 = 0$.

2. Let $f(x) = \sqrt{x+1}$. Find the third Taylor polynomial $P_3(x)$ about $x_0 = 0$. Use $P_3(x)$ to approximate $\sqrt{0.5}$, $\sqrt{0.75}$, $\sqrt{1.25}$, and $\sqrt{1.5}$ Determine the actual error of these approximations.

3. The Maclaurin series for $(1+x)^n$ is also known as the binomial series. It states that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots, \qquad (x^2 < 1).$$

   Derive this series by computing a Taylor's for $(1+x)^n$ around $x = 0$. Note that it is not assumed that n is an integer. Give its particular form in summation notation for $n = \frac{1}{2}$. Use this expression to compute $\sqrt{1.0001}$ correct to 15 decimal places.

4. Expand the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt,$$

   in a series by using the exponential series and integrating.

5. Read Chapters 2 and 3 in Michael Overton's book "Numerical Computing with IEEE Floating Point Arithmetic". Solve problems 3.1, 3.2, 3.3, 3.4, 3.6, 3.8. These chapters can be downloaded from the university library's webpage.