

AMSC/CMSC 460: Midterm 1

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.
- You may not use any books, notes, or calculators.
- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.
- Show all your work and explain everything you write.
- Exam time: 75 minutes
- Good luck!

Problems: (Each problem = 10 points)

1. (a) Write the number 35.35 in base 2. (Compute the first 10 digits after the binary point).
- (b) Explain how 35.35 can be represented as a floating point number on a 64-bit computer.
- (c) Explain how 35.35 can be stored with a fixed point representation on a 64-bit computer. What are the advantages of a floating point representation over a fixed point representation?
- (d) Explain two approaches for representing the (negative) number -35 on a computer with a 64-bit word.

2. Consider the following matrix A , and its inverse A^{-1} :

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 3 \\ 0 & 2 & -1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -1/6 & 1/6 & 2/3 \\ -1/3 & 1/3 & 1/3 \end{pmatrix}$$

- (a) Compute the condition number of A in the infinity norm.
 - (b) Find an LU decomposition of A where L is a unit lower triangular matrix.
 - (c) Use the LU decomposition that you found, to solve $Ax = b$ with $b = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$.
3. Let $f(x) = e^{-x} - x^2$.
 - (a) Prove that there exists at least one point $x^* \in [0, 10]$ for which $f(x^*) = 0$.
 - (b) Starting from $x_0 = 0$, use Newton's method to compute two approximations x_1 and x_2 for a root of $f(x)$.
 - (c) Starting from $x_0 = 0$ and $x_1 = 1$, compute one iteration of the secant method for the given function $f(x)$.

4. Let $f(x) = \cos(\pi x)$.

Let $x_0 = -1, x_1 = 0, x_2 = 1$, and let $y_j = f(x_j)$ for $j = 0, 1, 2$.

- (a) Write Newton's form for the interpolation polynomial, $P_2(x)$, that interpolates the data at the three given points.
- (b) Write Lagrange's form for the interpolation polynomial, $P_2(x)$, that interpolates the data at the three given points.
- (c) Verify that the answers to parts (a) and (b) are identical. Explain the advantages of Newton's form over Lagrange's form.
- (d) Write the Lagrange form of the polynomial $p_3(x)$, that interpolated $f(x) = \sin(\pi x)$ at the four points: $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 1/2$.