AMSC/CMSC 460: HW #5
Due: Tuesday 3/6/18 (in class)

Please submit the solution to at least one problem in LaTeX.
Problems 1–5 should be solved by hand. Problem 6 should be solved with Matlab.

1. Find the Lagrange and Newton forms of the interpolating polynomials for the following sets of data. Write both polynomials in the form $a+bx+cx^2$ in order to verify that they are identical.

   (a)

x	2	0	1
$f(x)$	10	7	15

   (b)

x	$-\sqrt{2/7}$	0	$\sqrt{1/7}$
$f(x)$	$f(-\sqrt{2/7})$	$f(0)$	$f(\sqrt{1/7})$

2. Find a polynomial that interpolates the following values

x	-4	3	2	5
$f(x)$	-3	-10	-7	0

3. Write the polynomial interpolating the function $f(x) = \exp(-x)$ at $x_0 = -1$, $x_1 = 0$, and $x_2 = 1$ (a) in Lagrange's form, (b) in Newton's form. Check that these polynomials coincide.

4. Write a polynomial interplant of $f(x) = \sin x$ in (a) Lagrange's and (b) Newton's form at the points $\{x_0, x_1, x_2, x_3, x_4\} = \{0, \pi/4, \pi/2, 3\pi/4, \pi\}$. You do not need to get rid of the factors of the form $(x - x_j)$, but all coefficients need to be evaluated. Then plot the graph of $\sin x$ and your interplant on the interval $[0, \pi]$.

5. The equation $x - 9^{-x} = 0$ has a solution in $[0, 1]$. Find the interpolation polynomial on $x_0 = 0, x_1 = 0.5, x_2 = 1$ for the function on the left side of the equation. By setting the interpolation polynomial equal to zero and solving the equation, find an approximate solution to the equation.

6. *To be done in Matlab: You should program this from scratch. Do not solve by using Matlab's internal interpolation routines.*

   Find interpolation polynomials in the Newton's form for each of the following three functions:
   (i) e^x on $[-3, 3]$,
   (ii) $\max\{0, |x| - 1\}$ on $[-2, 2]$.

   Use N equally spaced interpolation points, for $N = 3, 5, 10, 20$. Evaluate the interpolation polynomial at $10N$ equally spaced points. Plot the graphs of the functions and the interpolation polynomials for them for different values of N. Plot the graph of the maximal error of interpolation as a function of N for each of these functions. Write a summary of your observations.