AMSC/CMSC 460: HW #9
Due: Tuesday 4/26/18 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Using Taylor expansions, verify that the following two formulas approximate the third derivative. Find the error terms.

$$f'''(x) \approx \frac{1}{h^3}[f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$

$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

2. Using Taylor expansions, derive the error term for the formula

$$f''(x) \approx \frac{1}{h^2}[f(x) - 2f(x+h) + f(x+2h)].$$

3. Using the method of undetermined coefficients, establish the most accurate formula of the form

$$f'(x) \approx Af(x-h) + Bf(x+h) + Cf(x+2h) + Df(x+3h).$$

4. Using the method of undetermined coefficients, establish the most accurate formula of the form

$$f''(x) \approx Af(x) + Bf(x+h) + Cf(x+2h) + Df(x+3h).$$

5. Use the values of $f(x)$ at $x-3h, x-h, x+h, x+3h$ to obtain the most accurate approximation of $f'(x)$.

6. Interpolate the values of $f(x)$ at x_0-h, x_0, x_0+2h. Use the interpolant to find an approximation for $f'(x_0 + h/2)$.

7. Interpolate the values of $f(x)$ at x_0-h, x_0, x_0+h. Use the interpolant to find an approximation for $f'(x_0 - 2h)$. Note that the approach is still valid even though this point is outside of the interval $[x_0 - h, x_0 + h]$.