AMSC/CMSC 460: Midterm 2

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Read carefully the following instructions:

- Write your name & student ID on the exam book and sign it.

- You may <u>not</u> use any books, notes, or calculators.

- Solve all problems. Answer all problems after carefully reading them. Start every problem on a new page.

- Show all your work and explain everything you write.

- Exam time: 75 minutes

- Good luck!

Problems:

1. **(10 points)** Using Newton's form of the Hermite interpolation polynomial, find the polynomial of degree ≤ 3 that interpolates: $f(1) = 0$, $f'(1) = 2$, $f(2) = 1$, $f'(2) = 3$. (Compute explicitly all the divided differences).

2. Let $w(x) = 1$, $\forall x \in [-3, 2]$.

   (a) **(10 points)** Find the first two orthogonal polynomials with respect to the inner product

   $$\langle f(x), g(x) \rangle_w = \int_{-3}^{2} f(x)g(x)w(x)dx.$$

   (b) **(10 points)** Normalize the polynomials you found in part (a).

   (c) **(10 points)** Find the polynomial of degree 0, $Q_0(x)$, that minimizes

   $$\int_{-3}^{2} (e^x - Q_0(x))^2 dx.$$

3. Consider the following three data points: $(-\pi, 0)$, $(0, 0)$, $(\pi/2, 1)$.

   (a) **(10 points)** Write the Lagrange form of the quadratic polynomial that interpolates the given data.

   (b) **(10 points)** Repeat part (a) with Newton's form. Compute all the divided differences.

   (c) **(5 points)** Assuming that the given data points were sampled from $f(x) = \sin(x)$, find an expression for the interpolation error.